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A

NEW AND COMPLETE SYSTEM

OF

ARITHMETICK,

COMPOSED FOR THE

USE OF THE CITIZENS OF THE UNITED STATES.

BY NICOLAS PIKE, A. M. A. A. S.

Seventh Edition:

Abridged for the Use of Schools, (under the Direction of the Author) from the Third Octavo Edition, *which was revised, corrected, and improved, and more particularly adapted to the Federal Currency,*

BY NATHANIEL LORD, A. M.

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BE IT REMEMBERED, that on the ſeventh day of November, in the the thirty fourth Year of the Independence of the United States of America, THOMAS & ANDREWS of the ſaid Diſtrict, have depoſited in this Office the Title of a Book, the Right whereof they claim as proprietors in the Words following, *to wit*:—

"A new and complete ſyſtem of Arithmetick, compoſed for the uſe of the citizens of the United States. By Nicolas Pike, A. M. A. A. S. Seventh edition, abridged for the uſe of ſchools, (under the direction of the author) from the third octavo edition, which was reviſed, corrected and improved, and more particularly adapted to the federal currency, By Nathaniel Lord, A. M."

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WM. S. SHAW,
Clerk of the Diſtrict of Maſſachuſetts.

PREFACE

TO THE THIRD OCTAVO EDITION, FROM WHICH THIS ABRIDGMENT IS MADE.

THE demand for this work ſtill continuing, notwithſtanding the publication of other works on Arithmetick and the higher branches of the Mathematicks, is evidence of its intrinſick merit, and has induced the Proprietors of the copy-right to preſent the publick with a new and improved Edition.

Application was made to the Author, requeſting him to reviſe and improve the work for a new Edition; but he declined on account of want of health, and the Gentleman, whom we employed, was engaged by the Author's conſent, and improved and corrected the work agreeably to his direction and advice.

The moſt important improvement in this Edition, is the introduction of examples in the Federal Currency under each rule; and while this was conſidered neceſſary, in order to extend the knowledge and uſe of that currency, it was thought important not to omit examples in pounds, ſhillings and pence, which are, and will continue to be, the baſis of many arithmetical queſtions; and therefore an acquaintance with them will always be uſeful.

Mr. NATHANIEL LORD, 3d, of Ipſwich, the Gentleman employed to correct and improve the work, has beſtowed much attention upon it, and has received from Mr. PIKE all the information and advice he deſired. The manner in which Mr. LORD has executed the taſk entruſted to him, will, we hope, gain additional reputation for the work, and entitle him to the thanks of the publick.

THOMAS & ANDREWS.

Boſton, April, 1808.

EXPLANATION OF THE CHARACTERS

USED IN THIS TREATISE.

$=$	THE ſign of equality: as, 12 pence $= 1$ ſhilling, ſignifies that 12 pence are equal to one ſhilling; and, in general, that whatever precedes it is equal to what follows.
$+$	The ſign of Addition: as, $5+5=10$, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.
$-$	The ſign of Subtraction: as, $12-4=8$, that is, 12 leſſened by 4 is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 leſs 4 equal to 8.
$\times$	The ſign of Multiplication: as, $6\times5=30$, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.
$\div$ or 6)30(5	The ſign of Diviſion: as, $30\div5=6$, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.
$\frac{875}{25}$	Numbers placed fraction-wiſe do alſo denote diviſion, the numerator, or upper number being the dividend, and the denominator or lower number, the diviſor; thus, $\frac{875}{25}$ is the ſame as $875\div25=35$.
$: \; :: \; :$	The ſign of proportion, thus, $2 : 4 :: 8 : 16$, that is, as 2 is to 4 ſo is 8 to 16.
∺	Signifies Geometrical Progreſſion.
$\overline{9-2}+6=13$	Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line atop (called a *Vinculum*) connects all the numbers over which it is drawn.
$12-\overline{3+5}=4$	Signifies that the ſum of 3 and 5 taken from 12 leaves or is equal to 4.

$\overline{\quad}\vert^{2}$	Signifies the ſecond power, or Square.
$\overline{\quad}\vert^{3}$	Signifies the third power, or Cube.
$\sqrt{}$ or $\overline{\quad}\vert^{\frac{1}{2}}$	Prefixed to any number or quantity, ſignifies that the ſquare root of that number is required. It likewiſe (as alſo the character for any other root) ſtands for the expreſſion of the root of that number or quantity to which it is prefixed. As $\sqrt{36}=6$, and $\sqrt{108+36}=12$, or $\overline{36}\vert^{\frac{1}{2}}=6$, &c.
$\sqrt[3]{\quad}$ or $\vert^{\frac{1}{3}}$	Prefixed to any number, ſignifies that the cube root of that number is required or expreſſed. As $\sqrt[3]{216}=6$, and $\sqrt[3]{513+216} = 9$, &c.—or $\overline{216}\vert^{\frac{1}{3}}=6$, &c.

ARITHMETICK.

ARITHMETICK is the Art or Science of computing by numbers, and is compriſed under five principal or fundamental Rules, viz. *Notation*, or *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*.

NUMERATION

Teaches the different value of figures by their different places, and to read or write any ſum or number by theſe ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. 0 is called a cypher, and all the reſt are called figures or digits.

2. Beſides the ſimple value of figures as above noted, they have each a local value, according to the following law; viz. In a combination of figures, reckoning from right to left, the figure, in the firſt place, repreſents its primitive ſimple value; that in the ſecond place, ten times its ſimple value, and ſo on; the value of the figure, in each ſucceeding place, being ten times the value of it, in that immediately preceding it.

3. The values of the places are eſtimated according to their order: the firſt is denominated the place of units; the ſecond, tens; the third, hundreds; and ſo on, as in the Table. Thus, in the number 3467: 7, in the firſt place, ſignifies only ſeven; 6, in the ſecond place, ſignifies 6 tens, or ſixty; 4, in the third place, four hundred; 3, in the fourth place, three thouſand; and the whole, taken together, is read thus; three thouſand four hundred and ſixty ſeven.

4 A cypher, though of no ſignification itſelf, yet poſſeſſes a place, and, when ſet on the right hand of figures,

in whole numbers, increaſes their value in he ſame tenfold proportion; thus, 9 ſignifies only nine; but, if a cypher be placed on its right hand, thus, 90, it then becomes ninety.

To enumerate any parcel of figures, obſerve the following RULE.

Firſt. Commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the ſimple value of each figure, join the name of its place, beginning at the left hand and reading towards the right. *More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c half periods, and the figure over ſuch dot will, univerſally, have the name of thouſands.—2. Place the figures 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c period: Theſe indices will then ſhow the number of times the millions are involved—the figure under 1 bearing the name of millions, that under 2, the name of billions, (or millions of millions) that under 3, trillions, (or millions of millions of millions.)

EXAMPLE.

Sextill.	Quintill.	Quatrill.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x t.c.x.t.
6	5	4	3	2	1	
913,208;	000,341;	620,057;	219,356;	8[illegible]9,379;	120,406;	129,763
Thouſands	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*

NOTE 1. Billions is ſubſtituted for millions of millions; Trillions, for millions of millions of millions; Quatrillions for millions of millions of millions of millions.

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. anſwer to millions ſo often involved as their indices reſpectively denote.

NOTE 2. The right hand figure of each half period has the place of units, of that half period; the middle one, that of tens, and the left hand one, that of hundreds.

APPLICATION.

Write down, in proper figures, the following numbers.

Fifteen	15
Two hundred and ſeventy nine	279
Three thouſand four hundred and three	3403
Thirty ſeven thouſand five hundred and ſixty ſeven	37567
Four hundred one thouſand and twenty eight	401028
Nine millions ſeventy two thouſand and two hundred	9072200
Fifty five millions three hundred nine thouſand and nine	55309009
Eight hundred millions forty four thouſand and fifty five	800044055
Two thouſand five hundred and forty three millions four hundred and thirty one thouſand ſeven hundred and two	2543431702

Write down, in words at length, the following numbers.

8
17
129
437
3010
76506
709040
879096
4091875
3476194
84094007
690748591
7584397647
49163189186
500098422700

Notation by Roman Letters.

I. One.
II. Two.
III. Three.
IV. Four.
V. Five.
VI. Six.
VII. Seven.
VIII. Eight.
IX. Nine.
X. Ten.
XI. Eleven.
XII. Twelve.
XIII. Thirteen
XIV. Fourteen.
XV. Fifteen.
XVI Sixteen.
XVII. Seventeen.
XVIII. Eighteen.
XIX. Nineteen.
XX. Twenty.
XXX. Thirty.
XL. Forty.
L. Fifty.
LX. Sixty.
LXX. Seventy.
LXXX. Eighty.
XC. Ninety.
C. Hundred.
CC. Two Hundred.
CCC. Three Hundred.
CCCC. Four Hundred.
D. or IↃ. Five Hundred.
DC. Six Hundred.
DCC. Seven Hundred.
DCCC. Eight Hundred.
DCCCC. Nine Hundred.
M. or CIↃ. One Thouſand.
IↃↃ. Five Thouſand.
IↃↃↃ Fifty Thouſand.
IↃↃↃIↃↃↃ. Five Hundred Thouſ.
MDCCCIX. One Thouſand Eight Hundred and Nine.

A leſs literal number, placed after a greater, always augments the value of the greater; if put before, it diminiſhes it. Thus, VI is 6; IV is 4; XI is 11; IX is 9, &c.

ADDITION

IS the putting together of two or more numbers, or ſums, to make them one total, or whole ſum.

SIMPLE ADDITION

Is the adding of ſeveral integers or whole numbers together, which are all of one kind, or ſort; as, 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or ſum total, is 39 pounds.

RULE.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units: After adding up every figure in that column, conſider how many tens are contained in their ſum, and placing the exceſs under the units, carry ſo many as you have tens, to the next column, of tens: Proceed in the ſame manner through every column or row, and ſet down the whole amount of the laſt row.

PROOF.

Begin at the top of the ſum, and reckon the figures downwards, in the ſame manner as they were added upwards, and, if it be right, this aggregate will be equal to the firſt. Or, cut off the upper line of figures, and find the amount of the reſt; then if the amount and upper line, when added, be equal to the ſum total, the work is ſuppoſed to be right.

ADDITION and SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column, and the other atop, and in the common angle of meeting, or at the right hand of the firſt, and under the ſecond, you will find the ſum—as, 5 and 8 is 13.

When you would ſubtract; find the number to be ſubtracted in the left hand column, run your eye along to the right hand till you find the number from which it is to be taken, and right over it, atop, you will find the difference—as 8 taken from 13 leaves 5.

EXAMPLES.

1.	2.	3.	4.	5.	6.
£.	lb.	Cwt.	Miles.	Yards.	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	59	789	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	69	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4566	2102865
4567890	4566	89064	4321234
5678109	333	654321	5682098
6789098	90	1234567	6543218

SUBTRACTION

TEACHES to take a leſs number from a greater, to find a third, ſhewing the inequality, exceſs or difference between the given numbers; and it is both ſimple and compound.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

RULE.

Place the larger number uppermoſt, and the leſs underneath, ſo that units may ſtand under units, tens under tens, &c. then drawing a line underneath, begin with the units, and ſubtract the lower from the upper figure, and ſet down the remainder; but if the lower figure be greater than the upper, borrow ten and ſubtract the lower figure therefrom: To this difference add the upper figure, which being ſet down, you muſt add one to the ten's place of the lower line, for that which you borrowed; and thus proceed through the whole.

PROOF.

In either ſimple or compound Subtraction, add the remainder and the leſs line together, whoſe ſum, if the work be right, will be equal to the greater line:—Or, ſubtract the remainder from the greater line, and the difference will be equal to the leſs.

EXAMPLES.

	1.	2.	3.	4.	5.	6.
	£	£	Miles.	Yards.	Feet.	Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.						
Proof.						

7.	8.	9.
100200300400500600700800900	10000	1000000
98076054032011023045067089	9999	1

MULTIPLICATION

MAY be accounted the moſt ſerviceable rule in Arithmetick. It teaches how to increaſe the greater of two numbers given, as often as there are units in the leſs; performs the work of many additions in the moſt compendious manner; brings numbers of great denominations into ſmall, as pounds into ſhillings, pence, or farthings, &c.; and, by knowing the value of one thing, we find the value of many.

It conſiſts of three parts.

1. The Multiplicand, or number given to be multiplied, and commonly the largeſt number

2. The Multiplier, or number to multiply by, commonly the leaſt number.

3. The Product is the reſult of the work, or the anſwer to the queſtion.

SIMPLE MULTIPLICATION

Is the multiplying of any two numbers together, without having regard to their ſignification; as, 7 times 8 is 56, &c.

MULTIPLICATION and DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table for Multiplication;—Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or againſt your multiplier along at the right hand, and under your multiplicand, you will find the product or anſwer.

To learn it for Diviſion;—Find the diviſor in the left hand column, and run your eye along the row to the right hand until you find the dividend: then directly over the dividend, atop, you will find the quotient, ſhowing how often the diviſor is contained in the dividend.

CASE I.

When the multiplier is not more than 12.—Always placing the greateſt number uppermoſt, ſet the multiplier underneath, units under units, &c. and begin as the Table directs, ſetting down the unit figure under units, and carrying the tens to the next place, in all reſpects as in ſimple addition.

PROOF.

Multiply the multiplier by the multiplicand.

EXAMPLES.

1.	2.	3.	4.
37934	769308	4980076	763896
2	3	4	5
Prod.			

5.	6.	7.	8.
67589	503764	5918295	9164785
6	7	8	9

9.	10.	11.
4879567	5864794	8583478646
10	11	12

CASE II.

When the multiplier is more than 12.—Multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the firſt figure of each product exactly under its multiplier: Laſtly, add theſe ſeveral products together, in the ſame order, as they ſtand, and their ſum will be their total product.

EXAMPLES.

	1.	2.	3.
	6357534	8324629	46293843
	47	59	76
	44502738		
	25430136		
Prod.	298804098		

4.	5.	6.
647906	760483	91867584
4873	9152	6875
3157245938	6959940416	631589640000

CASE III.

When the multiplier is a compoſite number, that is, *when it is produced by the multiplication of any two numbers in the*

Table—Multiply the multiplicand by one of thoſe figures firſt, and that product by the other; and the laſt product will be the total required.

EXAMPLES.

1.	2.	3.
Multiply 59375 by 35	39187 by 48	91632 by 56

7×5=35 7

415625

5

2078125

CASE IV.

When there are cyphers on the right hand of either the multiplicand, or multiplier, or both—Neglect thoſe cyphers: then place the ſignificant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	8836301500000

CASE V.

To multiply by 10, 100, 1000, *&c.*—Set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.*

---

* This is evident from the nature of numbers, ſince every cypher annexed to the right of a number increaſes the value of that number in a tenfold proportion.

Examples.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.				

DIVISION.

TEACHES to ſeparate any number, or quantity given, into any number of parts aſſigned; or to find how often one number is contained in another; or from any two numbers given to find a third, which ſhall conſiſt of ſo many units, as the one of thoſe given numbers is comprehended in the other; and is a conciſe way of performing ſeveral ſubtractions.

There are four principal parts to be noticed in Diviſion, viz.

1. The Dividend, or number given to be divided.
2. The Diviſor, or number given to divide by.
3. The Quotient, or anſwer to the queſtion, which ſhows how often the diviſor is contained in the dividend.
4. The remainder (which is always leſs than the diviſor, and of the ſame name with the dividend) is very uncertain, as there is ſometimes a remainder, and ſometimes none.

Diviſion is both ſimple and compound.

Proof.

Multiply the diviſor and quotient together, and add the remainder, if there be any, to the product: If the work be right, that ſum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values; as, 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.

CASE I.

Rule.

Firſt, ſeek how many times the diviſor is contained in a competent number of the firſt figures of the dividend; when found, place the figure in the quotient; multiply the diviſor by this quotient figure; place the product under the left hand figures of the dividend; then ſubtract it therefrom, and bring down the next figure of the dividend to the right hand of the remainder: If, when you have brought down a figure to the remainder, it is ſtill leſs than the diviſor, a cypher muſt be placed in the quotient, and another figure be brought down; after which you muſt ſeek, multiply, and ſubtract, till you have brought down every figure of the dividend.*

Examples.

Diviſor. Divid. Quo.
3)175817(58605
15
—
25
24
—
18
18
—
17
15
—
2 Remainder.

Proof.
58605 Quotient.
×3 Diviſor +2
———
175817

---

* When there is no remainder to a diviſion, the quotient is the abſolute and perfect anſwer to the queſtion; but where there is a remainder, it may be obſerved, that it goes ſo much towards another time as it approaches the diviſor; thus, if the remainder be half the diviſor, it will go half of a time more and ſo on; in order, therefore, to complete the quotient, put the laſt remainder to the end of it, above a line, and the diviſor below it.

It is ſometimes difficult to find how often the diviſor may be had in the numbers of the ſeveral ſteps of the operation: The beſt way will be to find how often the firſt figure of the diviſor may be had in the firſt, or two firſt figures of the dividend, and the anſwer made leſs by one or two, is generally the figure wanted; but if, after ſubtracting the product of the diviſor and quotient from the dividend, the remainder be equal to, or exceed, the diviſor, the quotient figure

In this example, I find that 3, the diviſor, cannot be contained in the firſt figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the diviſor by it, ſetting the firſt figure of the multiplication under the 7 in the dividend, &c. I then ſubtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5; then, I enquire how often the diviſor 3 is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when finding I cannot have the diviſor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the firſt.

Obſerve, that in multiplying by 3, I add in the 2.

```
       2.                                    3.
29)153598(5296                   6493)91876375(14150
   145                                6493
   ---                                -----
    85                                26946
    58                                25972
    ---                               -----
    279                                9743
    261                                6493
    ---                                -----
     188                               32507
     174                               32465
     ---                               -----
      14                                 425
```

```
      4.                5.                 6.
28)503775(        35)197184(         85)994466(
      7.                8.                 9.
236)3798567(    3479)483956795(   5679)19647394(
              10.                      11.
   38473)119184693(           641976)9187642959(
                        12.
              5823789)791822376496(
```

---

muſt be increaſed accordingly; or if the product of the diviſor and quotient figure exceed the dividend, then the quotient figure muſt be proportionally leſſened.

13.

123456789)121932631112635269(

CASE II.

When there is one cypher or more at the right hand of the diviſor—It or they muſt be cut off; alſo cut off the ſame number of figures from the dividend, and then proceed as in caſe firſt: But the figures which were cut off from the dividend muſt be placed at the right hand of the remainder.

EXAMPLES.

1.

65|00)3794326|75(58374
325
544
520
243
195
482
455
276
260
1675 Remainder.

2.

5193|000)8937643|893(

3.

917|0)47658|3(

4.

875|000)91764789430|000(

5.

Quot. Rem.
1|0)9584|6

6.

Quot. Rem.
1|00)76495|80

7.

Quot. Rem.
1|000)93751839|462

NOTE. In dividing by 10, 100, 1000, &c. when you cut of as many figures from the dividend, as there are cyphers in the diviſor, your work is done; thoſe figures cut off at the right hand, are the remainder, and thoſe on the left, the quotient, as above.

CASE III.

SHORT DIVISION *is when the diviſor does not exceed* 12.

RULE.

Firſt, ſeek how often the diviſor can be had in the firſt figure, or figures of the dividend; which, when found, place in the quotient; then, *mentally*, multiply your diviſor by the figure placed in the quotient, and ſubtract the product from the like number of the left hand figures of your dividend, and the units which remain muſt be accounted ſo many tens, which you muſt ſuppoſe to ſtand at the left hand of the next figure in the dividend, and to be reckoned with it; then, ſeek how often you can have your diviſor in thoſe two figures; but, if nothing remain, you muſt then ſeek how often your diviſor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Diviſor. Dividend. 2. 3. 4.
2)71935 3)51903 5)633795 6)8471937

Quo. 35967—1

5. 6. 7.
8)5437846 9)45963784 11)91843756

CASE IV.

When the diviſor is ſuch a number, that any two, or more, figures in the table, being mutiplied together, will produce it, divide the given dividend by one of thoſe figures; the quotient, thence ariſing, by the other, and ſo on: and the laſt quotient will be the anſwer.*

---

* As the learner, at preſent, is ſuppoſed to be unacquainted with the nature of Fractions, and as the quotient is incomplete without the remainder—I ſhall here give a rule for finding the remainder without having recourſe to fractions.

RULE I.

Multiply the quotient by the diviſor; ſubtract the product from the dividend, and the reſult will be the true remainder.

The Rule, which is moſt commonly made uſe of, when the diviſor is a compoſite number, is,

RULE II.

Multiply the laſt remainder by the preceding diviſor, or laſt but one, and to the product add the preceding remainder; multiply this ſum by the next preceding diviſor, and to the product add the next preceding remainder; and ſo on till you have gone through all the diviſors and remainders, to the firſt.

EXAMPLES.

Firſt Method.	Second Method.
9)196473	8)196473
8)21830	9)24559
Quo. 2728—57	Quo. 2728—57

In the *firſt* operation, in dividing by 9, 3 remains, and by 8, 6 remains; which being the laſt remainder, I multiply it by the firſt diviſor 9, and add in the firſt remainder 3, and they make 57, the true remainder. In the *ſecond* method, dividing by 8, 1 remains and by 9, 7 remains; I thereſore multiply 7, the laſt remainder, by 8, adding in the 1, and they make 57, as before.

2.	3.	4.
25)167835	54)93738764	132)38473692

TABLES IN COMPOUND ADDITION.

1. FEDERAL MONEY.*

			marked
10 Mills	make 1	Cent	m. c.
10 Cents		Dime	d.
10 Dimes		Dollar	D.
10 Dollars		Eagle	E.

Mills
10= 1 Cent
100= 10= 1 dime.
1000= 100= 10= 1 dollar.
10000=1000=100=10=1 eagle.

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* It may be proper to introduce here an account of the Federal Money, as ſettled by Congreſs, the 8th of Auguſt, 1786, when it was *Reſolved*,

"That the ſtandard of the United States of America, for gold and ſilver, ſhall be eleven parts fine and one part alloy.

"That the Money Unit of the United States (being by the Reſolve of Congreſs of the 6th of July, 1785, a Dollar) ſhall contain of fine ſilver $375\frac{64}{100}$ grains.

2. ENGLISH MONEY.

4	Farthings	make 1	Penny	marked	qrs. d.
12	Pence		Shilling		s.
20	Shillings		Pound		£ or l.

Farthings

4 = 1 Penny
48 = 12= 1 Shilling
960 = 240=20=1 Pound.

" That the money of account, to correſpond with the diviſion of coins, agreeably to the above Reſolve, proceed in a decimal ratio, agreeably to the forms and manner following, viz.

" Mill, the loweſt money of account, of which 1000 ſhall be equal to the federal dollar, or money unit, 0·001

" Cent, the higheſt copper piece, of which 100 ſhall be equal to the federal dollar, 0·010

" Dime, the loweſt ſilver coin, of which 10 ſhall be equal to the dollar, 0·100

" Dollar, the higheſt ſilver coin, 1·000

" That betwixt the dollar and the loweſt copper coin, as fixed by a reſolve of Congreſs of the 6th of July, 1785, there ſhall be *three* ſilver coins, and *one* copper coin.

" That the ſilver coins ſhall be as follow : One coin containing $187\frac{82}{100}$ grains of fine ſilver, to be called *Half Dollar* : One coin containing $75\frac{128}{1000}$ grains of fine ſilver, to be called a *Double Dime* : And one coin containing $37\frac{564}{1000}$ grains of fine ſilver, to be called a *Dime*.

" That the two copper coins ſhall be as follow : One equal to the one hundredth part of the federal dollar, to be called a *Cent* : and one equal to the two hundredth part of the federal dollar, to be called a *Half Cent*.

" That 2¼ pounds Avoirdupois weight of copper ſhall conſtitute 100 Cents.

" That there ſhall be two gold coins : One containing $246\frac{268}{1000}$ grains of fine gold, equal to 10 dollars, to be ſtamped with the impreſſion of the American Eagle, and to be called an *Eagle* : One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars, to be ſtamped in like manner, and to be called a *Half Eagle*.

" That the mint price of one pound Troy weight of uncoined ſilver, eleven parts fine and one part alloy, ſhall be 9 dollars, 9 dimes, and 2 cents.

" That the mint price of one pound Troy weight of uncoined gold, eleven parts fine and one part alloy, ſhall be 209 dollars, 7 dimes, and 7 cents."

Pence Tables.

d.		s.	d.	d.		s.	d.
20	=	1	8	120	=	10	0
30	=	2	6	130	=	10	10
40	=	3	4	140	=	11	8
50	=	4	2	150	=	12	6
60	=	5	0	160	=	13	4
70	=	5	10	170	=	14	2
80	=	6	8	180	=	15	0
90	=	7	6	190	=	15	10
100	=	8	4	200	=	16	8
110	=	9	2	240	=	20	0

s.		d.	s.		d.
1	=	12	11	=	132
2	=	24	12	=	144
3	=	36	13	=	156
4	=	48	14	=	168
5	=	60	15	=	180
6	=	72	16	=	192
7	=	84	17	=	204
8	=	96	18	=	216
9	=	108	19	=	228
10	=	120	20	=	240

3. Troy Weight.*

				Marked
24	Grains	make 1	Pennyweight,	grs. pwt.
20	Pennyweights		Ounce	oz.
12	Ounces.		Pound	℔ or lb.

4. Avoirdupois Weight.†

					marked	
16	Drams	make	1	Ounce	dr.	oz.
16	Ounces		1	Pound		lb.
28	Pounds		¼	of a Hundred weight		qr.
4	Quarters		1	Hund. weight, or 112lbs.		Cwt.
20	Hundred weight		1	Ton.		T.

* By this weight are weighed Gold, Silver, Jewels, Electuaries, and all Liquors

Note 175 Troy ounces are preciſely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1lb Troy=5760 grains, and 1lb Avoirdupois = 7000 grains.

† By Avoirdupois are weighed all coarſe and droſſy goods, grocery and chandlery wares; bread, and all metals, except Gold and Silver.

5. APOTHECARIES' WEIGHT.[*]

20	Grains	make 1	Scruple	marked	gr. ℈
3	Scruples		Dram		ʒ
8	Drams		Ounce		℥
12	Ounces		Pound		lb.

6. CLOTH MEASURE.

2¼	Inches	make 1	Nail	marked in. na.
4	Nails, or 9 Inches		Quarter of a Yard	qr.
4	Quarters, or 36 In.		Yard	yd.
3	Quarters, or 27 In.		Ell Flemiſh	E. Fl.
5	Quarters, or 45 In.		Ell Engliſh	E. E.
6	Quarters, or 54 In.		Ell French	E. Fr.

7. LONG MEASURE.[†]

3	Barley corns	make 1	Inch	marked bar. in.
12	Inches		Foot	ft.
3	Feet		Yard	yd.
5½	Yards or 16½ ft.		Rod, Perch or Pole	pol.
40	Poles		Furlong	fur.
8	Furlongs		Mile	mile.
69½	Statute Miles, nearly		Degree of a great Circle	deg.
360	Degrees		A great Circle of the Earth.	

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[*] All the weights now uſed by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy, are the ſame, only differently divided and ſubdivided.

[†] The uſe of Long Meaſure is to meaſure the diſtance of places or any other thing, where length is conſidered without regard to breadth.

NOTE. 60 geometrical miles make a degree—4 inches a hand—5 feet a geometrical pace, 6 points make 1 line, 12 lines an inch, 12 inches a foot and 6 feet one French toiſe, or fathom, equal to 6 feet 4 inches, 8·812875 lines Engliſh meaſure. 1 Engliſh foot equal to 11 inches 3·1454 lines French. 66 feet, or 4 poles make a Gunter's chain. 3 miles make a league.

Or, in meaſuring Diſtances,

$7\frac{92}{100}$	Inches	make 1	Link
25	Links		Pole
100	Links		Chain
10	Chains		Furlong
8	Furlongs		Mile

8. TIME.*

60	Seconds	make 1	Minute	marked s.m.
60	Minutes		Hour	h.
24	Hours		Day	d.
7	Days		Week	w.
4	Weeks		Month	m.
13	Months 1 day & 6 hours		Julian Year.	yr.

9. LAND. OF SQUARE MEASURE.

144	Inches	make 1	Square Foot
9	Feet		—— Yard
$30\frac{1}{4}$	Yards, or $272\frac{1}{4}$ Feet		—— Pole
40	Poles		—— Rood
4	Roods, or 160 Rods, or 4840 Yards		—— Acre
640	Acres		—— Mile

10. SOLID MEASURE.†

1728	Inches	make 1	Foot
27	Feet		Yard
40	Feet of round timber, or 50 Feet of hewn timber,		Ton or Load
128	Solid Feet, i. e. 8 in length, 4 in breadth, and 4 in height		Cord of Wood

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* By the calendar, the year is divided in the following manner:

Thirty days hath September, April, June, and November;
Febuary twenty eight alone, and all the reſt have thirty one.

When you can divide the year of our Lord by 4, without any remainder, it is then Biſſextile, or Leap Year, in which February has 29 days.

† By Solid Meaſure are meaſured all things that have length, breadth, and depth.

11. WINE MEASURE.*

2 Pints	make 1	Quart	marked pt. qt.
4 Quarts		Gallon	gal.
10 Gallons		Anchor of Brandy	anc.
18 Gallons		Runlet	run.
31½ Gallons		Half Hogſhead	½hhd.
42 Gallons		Tierce	tier.
63 Gallons		Hogſhead	hhd.
2 Hogſheads		Pipe or Butt	P. or B.
2 Pipes		Tun	Tun.

12. ALE and BEER MEASURE.†

2 Pints	make 1	Quart	marked pt. qt.
4 Quarts		Gallon	gal.
8 Gallons		Firkin of Ale in London	A.fir.
8½ Gallons		Firkin of Ale or Beer	
9 Gallons		Firkin of Beer in Lon.	B.fir.
2 Firkins		Kilderkin	kil.
2 Kilderkins		Barrel	bar.
1½ Barrel, or 45 Gal.		Hogſhead of Beer	hhd.
2 Barrels		Puncheon	pun.
3 Barrels, or 2 Hhds.		Butt	butt.

13. DRY MEASURE.‡

2 Pints	make 1	Quart	marked pt. qt.
2 Quarts		Pottle	pot.
2 Pottles		Gallon	gal.
2 Gallons		Peck	pk.
4 Pecks		Buſhel	bu.
2 Buſhels		Strike	ſtr.
2 Strikes		Coom	co.
2 Cooms		Quarter	qr.
4 Quarters		Chaldron	ch.
4½ Quarters		Chaldron in London	
5 Quarters		Wey	wey.
2 Weys		Laſt	laſt.

* All Brandies, Spirits, Perry, Cider, Mead, Vinegar, Honey, and Oil, are meaſured by Wine Meaſure; Honey is, commonly, ſold by the pound Avoirdupois.

† Milk is ſold by the Beer quart.

‡ This meaſure is applied to all dry goods, as Corn, Seed, Fruits, Roots, Salt, Sand, Oyſters, and Coals.

A Wincheſter buſhel is 18½ inches diameter, and 8 inches dee

COMPOUND ADDITION

Is the adding of ſeveral numbers together, having different denominations, as, Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

Rule.

1. Place the numbers, ſo that thoſe of the ſame denomination may ſtand directly under each other.

2. Add the firſt column or denomination together as in whole numbers; then divide the ſum by as many of the ſame denomination, as make one of the next greater, ſetting down the remainder under the column added, and carry the quotient to the next ſuperiour denomination, continuing the ſame to the laſt, which add as in ſimple addition.

Examples.

1. Federal Money.

1.

E.	D.	d.	c.	m.
7	3	8	9	5
	2	1	2	5
9	0	0	5	
	3	6	2	5
7	1	4	0	8

2.

D.	c.	m.
49	18	7
25	32	1
93	7	5
13	25	
	97	2

3.

D.	c.	m.
375		
29	18	
7	12	5
199	18	7
30	01	

2. English Money.

1.

£	s.	d.
9	16	10
7	10	9
0	18	6
5	11	11

2.

£	s.	d.	qrs.
47	17	6	2
3	9	10	3
75	13	9	1
4	11	11	0

3.

£	s.	d.	qrs.
487	11	11	3
418	19	6	1
59	6	10	0
747	16	1	2

3. TROY WEIGHT.

1.

lb.	oz.	pwt.
767	10	17
39	6	9
417	11	16
935	9	17

2.

lb.	oz.	pwt.	gr.
649	11	19	20
32	9	6	5
841	10	11	19
473	9	17	23

3.

lb.	oz.	pwt.	gr
859	9	15	20
437	10	17	22
641	11	6	
738	9	12	18

4. AVOIRDUPOIS WEIGHT.

1.

lb.	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5

2.

T.	cwt.	qrs.	lb.
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19

3.

T.	cwt.	qrs.	lb.	oz.	dr.
91	17	2	25	13	15
19	9	0	17	10	12
14	13	2	0	9	11
47	11	3	19	14	0

5. APOTHECARIES' WEIGHT.

1.

ʒ	℈	gr.
9	1	17
3	2	19
6	1	17
4	0	6

2.

℥	ʒ	℈	gr.
10	7	2	19
6	3	0	12
7	6	1	17
9	5	2	12

3.

lb.	℥	ʒ	℈	gr.
12	11	6	1	15
4	9	1	0	12
91	10	7	2	16
4	8	1	2	19

6. CLOTH MEASURE.

1.

Yd.	qr.	n.
76	2	3
3	3	1
42	3	3
57	2	2

2.

E.E.	qr.	n.
91	3	2
49	4	3
6	2	3
84	4	1

3.

E.Fl.	qr.	n.
75	2	1
7	1	3
84	0	2
76	2	3

7. LONG MEASURE.

1.

Pol.	ft.	in.
12	11	10
9	10	9
8	12	11
7	15	6

2.

Mil.	fur.	pol.
9	7	36
7	3	19
4	1	24
6	5	12

3.

Deg.	m.	fu.	po.	ft.	in.	bc.
759	56	6	29	15	10	2
317	39	1	36	11	6	1
497	63	7	24	9	8	1
562	17	0	11	13	11	0

8. TIME.

1.

W.	d.	h.	m.	s.
3	6	22	57	42
1	5	19	31	28
2	3	17	9	15
3	0	9	17	58

2.

Y.	mo.	d.
19	10	19
7	9	27
4	8	16
1	11	14

3.

Y.	mo.	w.	d.	h.	m.	s.
57	11	3	6	23	29	55
4	8	1	1	19	45	38
29	9	2	3	17	18	19
46	10	2	5	11	50	13

9. LAND, OR SQUARE MEASURE.

1.

Pol.	ft.	in.
36	179	137
19	248	119
12	96	75
18	110	122

2.

Yd.	ft.	in.
28	7	119
9	3	75
29	6	120
4	8	12

3.

Acr.	roo.	pol.	ft.	in.
756	3	37	245	128
29	1	28	93	25
416	3	[illegible]	128	119
37	1	19	218	20

10. SOLID MEASURE.

1.

Ton.	ft.	in.
29	36	1229
12	19	64
18	11	917
19	8	1001

2.

Yd.	ft.	in.
75	22	1412
9	26	195
3	19	1091
28	15	1110

3.

Cord.	ft.	in.
37	119	1015
9	110	159
48	127	1071
8	111	956

11 Wine Measure.

1.

Tie.	gal.	qt.	pt.
37	39	3	1
9	17	2	1
4	28	0	0
32	19	1	1

2.

Hhd.	gal.	qt.	pt.
51	53	1	1
27	39	3	0
9	18	0	1
0	9	2	1

3.

Tun.	hhd.	gal.	qt.
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1

12. Ale and Beer Measure.

1.

A.B.	fir.	gal.
49	3	7
26	2	3
9	0	4
17	3	0

2.

B.B.	fir.	gal.
29	1	8
19	3	5
16	0	3
9	1	8

3.

Hhd.	gal.	qt.
379	53	3
19	0	1
121	37	2
467	19	1

13. Dry Measure.

1.

Qr.	bu.	p.	qt.
64	7	3	7
9	4	1	5
19	6	2	1
4	0	2	0

2.

Bu.	p.	qt.	pt.
37	2	5	1
19	3	7	0
16	2	0	1
5	1	6	1

3.

Ch.	bu.	p.	qt.
37	27	3	5
6	29	1	7
15	30	0	0
4	11	3	0

COMPOUND SUBTRACTION

Teaches to find the difference, inequality, or exceſs, between any two ſums of divers denominations.

Rule.

Place thoſe numbers under each other, which are of the ſame denomination, the leſs being below the greater;

begin with the leaſt denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; ſubtract it therefrom; and to the difference add the upper figure, remembering always, to add one to the next ſuperiour denomination, for that which you borrowed.

EXAMPLES.

1. FEDERAL MONEY.

	D.	c.	m.
From	39	15	5
Take	28	17	2

E.	D.	c.	m.
21	8	1	2
10	7	5	

D.	c.	m.
100		
48	87	5

	D.	c.
Borrowed	100	
Paid	20	18
Remains to pay		

	D.	c. m.
Lent	200	
Received	145	50
Due to me		

	D.	c.	m.
Borrowed	3000		
Paid	195		
at	1115	46	
ſeveral	247	37	5
times.	995	12	5
Paid in all			
Remains to pay			

	D.	c.	m.
Lent	7159	12	8
Received	245	37	5
at	3112	15	7
ſeveral	2000		
times.	1092	92	0
Received in all			
Remains due			

2. ENGLISH MONEY.

1.

	£.	s.	d.	qr.
Borrowed	349	15	6	1
Paid	195	11	8	1
Remains to pay	154	3	10	0

2.

	£.	s.	d.	q.
Lent	791	9	8	1
Received	107	16	4	2
Due me				

3.

	£.	s.	d.	q.
Borrowed	19372	12	9	0
Paid at	293	16	8	0
	74	9	7	7
ſundry	9413	11	0	1
	1994	0	10	3
times	3914	19	0	0
	1064	17	9	1
Paid in all				
Rem. to be pd.				

4.

	£.	s.	d.	q.
Lent	27109	5	8	3
Rec. at	5196	15	10	0
	384	16	6	2
ſeveral	4187	18	11	1
	6649	16	8	0
times	917	9	10	3
	3196	0	2	3
Rec. in all				
Rem. due				

3. TROY WEIGHT.

	1.				2.				3.			
	lb.	oz.	pwt.	gr.	lb.	oz.	pwt.	gr.	lb.	oz.	pwt.	gr.
Bought	749	5	13	16	189	8	12	10	543	3	9	13
Sold	96	9	19	13	148	4	16	19	179	1	15	18
Rem.												

4. AVOIRDUPOIS WEIGHT.

	1.			2.			3.					
	lb.	oz.	dr.	C.	qr.	lb.	T.	cwt.	qr.	lb.	oz.	dr.
Bought	7	9	12	8	2	13	9	11	3	17	5	12
Sold	3	12	9	4	1	15	3	12	1	19	10	9

5. APOTHECARIES' WEIGHT.

1.					2.					3.				
lb	℥	ʒ	℈	gr.	lb	℥	ʒ	℈	gr.	lb	℥	ʒ	℈	gr.
71	9	3	1	13	65	10	6	2	1	84	1	1	1	1
37	8	4	1	16	31	8	4	2	0	65	9	3	1	17

6. CLOTH MEASURE.

1.

Yd.	qr.	n.
35	1	2
19	1	3

2.

E. E.	qr.	n.
467	3	1
291	3	2

3.

E. Fl.	qr.	n.
765	1	3
149	2	1

4.

E. Fr.	qr.	n.
549	4	2
197	4	3

7. LONG MEASURE.

1.

Yd.	ft.	in.
28	2	10
17	2	11

2

Mil.	fur.	pol.
76	3	11
27	3	21

3.

Deg.	m.	fur.	p.	yd.	ft.	in.	bar.
38	41	3	29	2	1	7	2
19	35	5	31	3	1	9	1

8. TIME.

1.

Mo.	d.	h.	m.	s.
6	17	13	27	19
1	21	16	41	35

2.

Y.	mo.	d.
7	3	13
4	2	19

3.

Y.	mo.	w.	d.	h.	m.	s.
48	9	2	5	19	27	31
19	9	3	4	20	19	49

4. LAND OR SQUARE MEASURE.

1.

A.	R.	Pol.
29	1	10
24	1	25

2.

A.	R.	Pol.
29	3	19
17	1	36

3.

A.	R.	Pol.	ft.	in.
56	2	16	27	110
29	0	21	210	129

10. SOLID MEASURE.

1.

T.	ft.	in.
49	19	1100
38	36	1296

2.

Yds.	ft.	in.
79	11	917
17	25	1095

3.

Cords.	ft.	in.
349	97	1250
192	127	1349

11. WINE MEASURE.

1.

Hhd.	gal.	qt.	pt.
79	21	2	1
38	61	3	1

2.

Tier.	gal.	qt.
19	17	1
12	29	2

3.

Tun.	hhd.	gal.
532	1	19
197	1	47

12. ALE AND BEER MEASURE.

1.

A.B.	fir.	gal.	qt.
39	1	2	1
24	3	6	2

2.

B.B.	fir.	gal.	qt.	pt.
21	3	5	2	0
19	1	7	2	1

3.

Hhd.	gal.	qt.
769	17	1
391	42	3

13. DRY MEASURE.

1.

Qr.	bu.	pk.	qt.
56	2	2	1
39	3	1	2

2.

Bu.	pk.	qt.	pt.
91	1	3	2
29	2	1	1

3.

Chal.	bu.	pk.	qt.
39	12	2	1
24	25	3	2

PROBLEMS

Reſulting from a Compariſon *of the preceding Rules.*

PROB. 1. Having the ſum of two numbers and one of them given to find the other.

Rule. Subtract the given number from the given ſum, and the remainder will be the number required.

Let 288 be the ſum of two numbers, one of which is 115, the other is required.

From 288 the ſum,
Take 115 the given number.

Remains 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the leſs, given, to find the leſs.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other 198: What is the other?

From 325 the greater,
Take 198 the difference.

Rem. 127 the leſs.

Prob. 3. Having the leaſt of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the leſs number,
198 the difference.

Sum 325 the greater number required.

Prob. 4. Having the ſum and difference of two numbers given, to find thoſe numbers

Rule. To half the ſum add half the difference, and the ſum is the greater; and from half the ſum take half the difference, and the remainder is the leſs.—Or, From the ſum take the difference, and half the remainder is the leaſt: To the leaſt add the given difference, and the ſum is the greateſt.

What are thoſe two numbers, whoſe ſum is 48, and difference 14?

2)48 2)14

Half ſum =24 Half diff = 7

24+7=31 the greater; and 24—7=17 the leſs.

Or, 48—14÷2=17; and 17+14=31.

Prob. 5 Having the ſum of two numbers and the difference of their ſquares* given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the ſum of the numbers, and the quotient will be their difference: You will then have their ſum and difference, to find the numbers by Problem 4.

What two numbers are thoſe, whoſe ſum is 32, and the difference of whoſe ſquares is 256?

---

* The ſquare of a number is the product of it multiplied into itſelf.

Half ſum 16
Half diff. 4
——
Greater 20
——
Leſs 12

32)256(8 difference.
256
——

Prob. 6. Having the difference of two numbers, and the difference of their ſquares given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the difference of the numbers, and the quotient will be their ſum; then proceed by Problem 4.

What are thoſe two numbers, whoſe difference is 20, and the difference of whoſe ſquares is 2000?

20)2000(100 ſum. 50+10=60, the greater; and
50—10=40 the leſs.

Prob. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288, and one of them 8; I demand the other.

8)288
——
Ans. 36

Prob. 8. Having the dividend and quotient, to find the diviſor.

Rule. Divide the dividend by the quotient.

Cor. Hence we get another method of proving diviſion.

Given { 288 the dividend,
36 the quotient.
Required the diviſor.

36)288(8 diviſor.
288
——

Prob. 9. Having the diviſor and quotient given, to find the dividend.

Rule. Multiply them together.

Given { 8 the diviſor,
36 the quotient.
Required the dividend.

36
8
——
288 the divdend.

By a due conſideration and application of theſe Problems only, many queſtions may be reſolved in a ſhort and elegant manner, although ſome of them are generally ſuppoſed to belong to higher rules.

APPLICATION OF THE PRECEDING RULES.

1. The leaſt of two numbers is 19418, and the difference between them is 2384 : What is the greater, and ſum of both ?

19418+2384=21802, greater ; and 19418+21802=41220, ſum.

2. Suppoſe a man born in the year 1743 : When will he be 77 years of age ? 1743+77=1820, Ans.

3. What number is that, which being added to 19418, will make 21802 ? 21802—19418=2384, Ans.

4. General Waſhington was born in 1732 : What was his age in 1799 ? 1799—1732=67, Ans.

5. America was diſcovered by Columbus in 1492, and its independence was declared in 1776 : How many years elapſed between thoſe two eras ?

1776—1492=284, Ans.

6. The maſſacre at Boſton, by the Britiſh troops, happened March 5th, 1770, and the battle at Lexington, April 19th, 1775 : How long between ?

April 19, 1775—March 5, 1770=5y 1m. 14d. Ans.

7. General Burgoyne and his army were captured Oct. 17th, 1777, and Earl Cornwallis and his army, Oct. 19th, 1781 : What ſpace of time between ?

October 19, 1781—October 17, 1777=4y 2d. Ans.

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783 : How long did the war continue ?

Jan. 20, 1783—April 19, 1775=7y. 9m. 1d.

9. L, M, N, and O purchaſed a quantity of goods in partnerſhip ; L paid £12 10s, a dollar,* and a crown† piece ; M, 35s. ; N, 29s. 10d. ; and O, 79d. : What did the goods coſt ? Ans. £16 14 1.

10. A man borrowed, at different times, theſe ſeveral ſums, viz. £29 5 ; £18 17 6 ; £45 12 ; £98, 3 dollars, one crown piece, and a half : How much was he in debt ? Ans. £193 2 6.

---

* 6s. † 6s 8d

11. There are 4 numbers; the firſt 317, the ſecond 912, the third 1229, and the fourth as much as the other three, abating 97; What is the ſum of all? Ans. 4819.

12. Bought a quantity of Goods for £125 10s. paid for truckage 45s. for freight 79s. 6d. for duties 35s. 10d and my expenſes were 53s. 9d.: What did the goods ſtand me in? Ans. £136 4s. 1d.

13. A gentleman left his ſon £1725 more than his daughter, whoſe fortune was 15 thouſand 15 hundred and 15 pounds: What was the ſon's portion, and what did the whole eſtate amount to?

Ans. The ſon's fortune, £18240, and the whole eſtate £34755.

14 A merchant had ſix debtors, who, together, owed him £2917 10s. 6d. A, B, C, D, and E, owed him £1675 13s. 9d. of it: What was F's debt?

Ans. £1241 16s. 9d.

15. What is the difference between £1309 7s. 1d. and the amount of £345 13s. 4d. and £571 4s. 8d.?

Ans. £392 9s. 1d.

16 A merchant, at his firſt engaging in trade, owed £937 15s. he had in caſh £1755 3s. 6d. in goods £459 12s 3d. in good debts, £197 16s. and he cleared the firſt year £249 19s. 10d. What was the neat balance at the year's end? Ans. £1724 16s 7d.

17. What ſum of money muſt be divided between 12 men, ſo as that each may receive £155?

£155×12=£[illegible]60 Ans.

18. What number muſt I multiply by 9, that the product may be 675? 175÷9=75 Ans.

19. A privateer of 175 men took a prize which amounted to £59 per man, beſides the owner's half: What was the value of the prize? 175×59×2=£20650 Ans.

20. What is the difference between thrice five, and thirty, and thrice thirty five?

$\overline{35\times3}-\overline{5\times3+30}=60$, Ans.

21. The ſum of two numbers is 750; the leſs 248: What is their difference, product, and the ſquare of their difference?

750—248=502 the greater number, 502—248=254 difference, 502×248=124496 product, and 254×354=64516 ſquare of the difference.

22. What is the difference between ſix dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the leſs?

Ans. $\overline{6\times12\times12}-\overline{6\times12}=\underline{792}$ diff.

6×12×12×6×12=62208 product, and 6×12×12÷6×12=12, Quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their ſum and product are required.

Ans. 78×25=1950 the greater, $1950-\overline{15\times9}=1815$ the leſs. 1950+1815=3765 the ſum, and 1950×1815=3539250 the product.

24. A merchant began trade with £25327; for 6 years together, he cleared £1253 per annum; the next 5 years he cleared £1729 per annum; but, the laſt 4 years, had the misfortune to loſe £3019 per annum: What was he worth at the 15 years end? Ans. £29414.

25. If a man ſpend £192 in a year, what is that per calendar month; 192÷12=16l. Ans.

26. If the Federal debt, which is 42,000,000 dollars, be equally divided between the thirteen ſtates; what will be the ſhare of each? Ans. $3230769\frac{3}{13}$ dols.

27. If 9000 men march in a column of 750 deep; how many march abreaſt? 9000÷750=12, Ans.

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the ſame?

$\overline{3072\div32}-32=64$, Ans.

29. What number is that, which, being multiplied by 3589, will produce 92050672?

92050672÷3589=25648, Ans.

30. Suppoſe the quotient ariſing from the diviſion of two numbers to be 5379, and the diviſor 37625: What is the dividend, if the remainder come out 9357?

$\overline{32625\times5379+9357}=202394232$, Ans.

REDUCTION

TEACHES to bring, or exchange, numbers of one denomination to others of different denominations, retaining the ſame value.

It is of *two* ſorts, viz. Deſcending and Aſcending; the former of which is performed by Multiplication, and the latter by Diviſion.

REDUCTION DESCENDING.

RULE.

Multiply the higheſt denomination, given, by ſo many of the next leſs, as make one of that greater, and thus continue, till you have brought it down as low as your queſtion requires.

PROOF.

Change the order of the queſtion, and divide your laſt product by the laſt multiplier, and ſo on.

NOTE. From this rule and Caſe IV. of Simple Multiplication it appears, that Federal Money is reduced from higher to lower denominations by annexing as many cyphers as there are places from the denomination given to to that required; or, if the given ſum be of different denominations, by annexing the ſeveral figures of all the denominations in their order, and continuing with cyphers, if neceſſary, to the denomination required; or, what amounts to the ſame thing, by reading the whole number from the left to the required denomination, as one number in the required denomination.

EXAMPLES.

1. In 3 eagles 2 dollars, how many mills?
Ans. 32000m.
2. In 91 dollars 75 cents, how many cents?
Ans. 9175c.
3. In 50 eagles, how many dollars? Ans. 500D.
4. In 44 dollars 1 cent 4 mills, how many mills?
5. In 9 dollars 31 cents 7 mills, how many mills?
6. How many cents in 39 dollars 5 cents?
7. In 28 dollars 17 cents 5 mills, how many mills?

8. In £27 15s. 9d. 2qrs. how many farthings?

£ s. d. qrs.
27 15 9 2
Multiplied by 20 = ſhillings in a pound.

555 = ſhillings.
—— by 12 = pence in a ſhilling.

6669 = pence.
—— by 4 = farthings in a penny.

Ans. = 26678 farthings.

NOTE. In multiplying by 20, I added in the 15s.; by 12, the 9d.; and by 4, the 2qrs. which muſt always be done in like caſes.

To prove the above queſtion, change the order of it, and it will ſtand thus: In 26678 farthings, how many pounds?

4)26678
12)6669 2qrs.
2|0)55|5 9d.
Ans. £27 15 9 2

9. In £36 12s. 10d. 1qr. how many farthings?
Ans. 35177.

10. In £95 11s. 5d. 3qrs how many farthings?
Ans. 91751.

11. In £719 9s. 11d. how many half pence?
Ans. 345358.

12. In 29 guineas, at 28s. each, how many pence?
Ans. 9744.

13. In 37 piſtoles at 22s. how many ſhillings pence and farthings? Ans. 814s. 9768d 39072 farthings.

14. In 49 half johannes, at 48s. how many ſixpences?
Ans. 4704.

15. In 473 French crowns, at 6s. 8d. how many three pences? Ans. 12613$\frac{1}{3}$.

16 In 53 moidores, at 36s. how many ſhillings, pence and farthings? Ans. 1908s. 22896d. 91584qrs.

17. In £29 how many groats, threepences, pence and farthings?

Ans. 1740 groats, 2320 threepences, 6960d. 27840qrs.

18. Reduce 47 guineas and one fourth of a guinea into ſhillings, ſixpences, groats, three pences, twopences, pence and farthings.

Ans. 1323 ſhillings, 2646 ſix-pences, 3969 groats, 5292 three-pences, 7938 two-pences, 15876d. and 63504qrs.

REDUCTION ASCENDING.

Rule.

Divide the loweſt denomination given, by ſo many of that name, as make one of the next higher, and thus continue, till you have brought it into that denomination, which your queſtion requires.

Note. From this rule and the note under Caſe II. of Simple Diviſion, it appears, that Federal Money is reduced from lower to higher denominations by cutting off as many places as the given denomination ſtands to the right of that required; the figures cut off belonging to their reſpective denominations.

Examples.

1. How many eagles in 32000 mills? Ans. 3E. 2D.
2. In 9175 cents, how many dollars? Ans. 91D. 75c.
3. In 500 dollars, how many eagles? Ans. 50.
4. In 4414 mills, how many dimes?
5. In 9317 mills, how many dollars?
6. How many dollars in 28175 mills?
7. In 547325 farthings, how many pence, ſhillings, and pounds?

Farthings in a pennny = 4)547325

Pence in a ſhilling = 12)136831 1qr.

Shillings in a pound = 2|0)1140|2 7d.

£570 2s. 7d. 1qr.

Ans. 136831d.; 11402s.; 570l.

Note. The remainder is always of the ſame name as the dividend.

8. Bring 35177 farthings into pounds.

9. Bring 91751 farthings into pence, &c.

10. Bring 345338 half pence into pence, ſhillings, and pounds.

11. Reduce 9744 pence into guineas, at 28s. per guinea.

12. In 39072 farthings, how many piſtoles, at 22s. ?

13. In 4704 ſix-pences, how many half johannes ?

14. In 12613⅓ three-pences, how many French crowns, at 6s. 8d.

15. In 91584 farthings, how many moidores, at 36s. ?

16. In 27840 farthings, how many pence, three-pences, groats, ſhillings, and pounds ?

17. In 63504 farthings, how many pence, two-pences, three-pences, groats, ſix-pences, ſhillings, and guineas ?

Note. The preceding queſtion may ſerve as proofs to thoſe in Reduction deſcending.

REDUCTION DESCENDING and ASCENDING.

1. Money.

1. In £97 how many pence, and Engliſh or French crowns, at 6s. 8d.? Ans. 23280d. and 291 crowns.

2. In 519 Engliſh half crowns, how many pence and pounds? Ans. 20760d. and 86l. 10s.

3. In 735 French crowns, how many ſhillings and French guineas, at 26s. 8d.

Ans. 4900s. and 183 guineas, 20s.

4. In 5793 pence, how many farthings, pounds and piſtoles?

Ans. 23172qrs.; 24l. 2s. 9d.; and 21 piſtoles, 20s. 9d.

5. In 59 half joes, 37 moidores, 45 guineas, 63 piſtoles, 24 Engliſh crowns, and 19 dollars; how many

pounds, half joes, moidores, guineas, piſtoles, Engliſh crowns, dollars, ſhillings, pence and farthings?

Ans. 354l. 4s.; 147 half joes, 28s.; 196 moidores, 28s.; 253 guineas; 322 piſtoles; 1062 Engliſh crowns, 4s.; 1180 dollars, 4s.; 7084 ſhillings; 85008 pence, and 340032 farthings.

When it is required to know how many ſorts of coin, of different values, and of equal number, are contained in any number of another kind; reduce the ſeveral ſorts of coin into the loweſt denomination mentioned, and add them together for a diviſor; then reduce the money given into the ſame denomination for a dividend, and the quotient ariſing from the diviſion will be the number required.

Note. Obſerve the ſame direction in weights and meaſures.

1. In 275 half Johannes, how many moidores, guineas, piſtoles, dollars, ſhillings, and ſix-pences, of each the like number?

A moid. is 36s. = 72 ſixpences.
A guinea is 28s. = 56 do.
A piſtole is 22s. = 44 do.
A dollar is 6s. = 12 do.
One ſhilling = 2 do.
1 do.
Diviſor = 187 ſix-pences.

275 half joes.
48s. in a johan.
2200
1100
13200 ſhillings.
2 ſix-p. in a ſhill.
Dividend = 26400 ſixpences.

187)26400(141 of each, and 33 ſix-pences, or 16s. 6d. over, the anſwer.

2. A gentleman diſtributed 37l. 10s. between 4 perſons in the following manner, viz. that as often as the firſt had 20s. the ſecond ſhould have 15s. the third 10s. and the fourth 5s. What did each perſon receive?

Ans. The firſt man 15l.; ſecond 7l. 10s.; third 3l. 15s.

2. Troy Weight.

1. How many grains in a ſilver bowl, that weighs 3lb. 10 oz. 12pwt.?

lb. oz. pwt.
3 10 12
12 ounces in a pound.

46 ounces.
20 pennyweights in an ounce.

932 pennyweights.
24 grains in one pwt.

3728
1864

Proof 24)22368 grains, Ans.

2|0)93|2

12)46 12pwt.

3lb. 10 oz.

2. In 13 ingots of gold, each weighing 9oz. 5pwt. how many grain? Ans. 57720gr.

3. In 97397 grains, how many pounds? Ans 16lb. 10oz. 18pwt. 5gr.

4. How many rings, each weighing 5pwt. 7gr. may be made of 3lb. 5oz. 16pwt. 2gr. of gold? Ans. 158.

3. Avoirdupois Weight.

Cwt. qrs. lb. oz.
1. In 91 3 17 14 how many ounces?
4

367 quarters.
28

2943
735

10293 pounds.
16

61762
10293

164702 ounces.

Proof.
16)164702
28)10293 14oz.

4)367 17lb.

Cwt. 91 3qrs.

2. In 12 tons, 15cwt. 1qr. 19lb. 6oz. 13dr. how many drams? Ans. 7323500dr.

3. In 24lb. 11oz. 9dr. how many drams? Ans. 6329dr.

4. In 44800 pounds, how many drams and tons? Ans. 11468800dr. and 20 tons.

5. In 28 pounds avoirdupois, how many pounds Troy?

```
                                28lb.
                                 7000  grains in 1lb. avoir.
                                ------
grs. in 1lb. Tr. =576|0)19600|0(34lb.
                        1728
                        ----
                        2320
                        2304
                        ----
                          160
                           12
                        ------
                 576|0)192|0(0 oz.
                           20
                        ------
                 576|0)3840|0(6pwt.
                        3456
                        ------
                          3840
                            24
                        ------
                          1536
                          768
                        ------
                 576|0)9216|0(16gr.
                        576
                        ----
                        3456
                        3456
                        ----
```

6. In 47lb. 9oz. 13pwt. 17grs. Troy, how many pounds, avoirdupois?

lb. oz. pwt. gr.
47 9 13 17
12
573
20
11473
24
45899
22947
7|000)275|369(39lb.
21
65
63
2369
16
14214
2369
7|000)37|904(5oz.
35
2904
16
17424
2904
7|000)46|464(6$\frac{4464}{7000}$ drams.
42
4464

4. Apothecaries' Weight.

1. How many grains are there in 37lb. 6℥?

lb. ℥
37 6
12
---
450 ounces.
8

3600 drams.
3

10800 ſcruples.
20

216000 grains.

Proof.
2|0)21600|0
3)10800
8)3600
12)450
37lb. 6℥

2. In 9lb. 8℥ 1ʒ 2℈ 19gr. how many grains?
Ans. 55799gr.

3. In 55799 grains, how many pounds, &c.?
Ans. 9lb. 8oz. 1ʒ 2℈ 19gr.

5. Cloth Measure.

1. In 127 yards, how many quarters and nails?

Yds.
127
4
---
508 qrs.
4

Ans. 2032 nails.

Proof.
4)2032
4)508
127 yds.

2. In 9173 nails, how many yards?
Ans. 573yds. 1qr. 1n.

3. In 75 ells Engliſh, how many quarters and nails?
Ans. 375qrs. 1500n.

4. In 56 ells Flemiſh, how many quarters and nails?
Ans. 168qrs. 672n.

5. In 7248 nails, how many yards, ells Flemiſh, and ells Engliſh?
Ans. 453yds. 604 ells Flemiſh, 362 ells En. 2qrs.

6. In 19 pieces of cloth, each 15yds. 2qrs. how many yards, quarters, and nails?

Ans. 294yds. 2qrs.; 1178qrs. and 4712n.

6. Long Measure.

1. How many barley-corns will reach from Newburyport to Boſton, it being 43 miles?

Miles.	Proof.
43	3)8173440
8	
	12)2724480
344 furlongs.	
40	3)227040
13760 rods.	11)75680
5½	
	6880
68800	2
6880	
	4|0)1376|0
75680 yards.	
3	8)344
227040 feet.	43
12	
2724480 inches.	
3	
8173440 Ans.	

Here I divide by 11 and multiply the quotient by 2, becauſe twice 5½ is 11;—or, I might firſt have multiplied by 2, and then have divided the product by 11.

2. How many barley-corns will reach round the globe, it being 360 degrees? Ans. 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles? Ans. 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the diſtance from Newburyport to Cambridge, it being 42 miles?

Ans. 13440 times.

5. In 190080 inches, how many yards and leagues?
Ans. 5280yds. and 1 league.

7. TIME.

1. In 20 years, how many ſeconds?

```
   d.  h.
  365  6 in a year.
   24
 ----
 1466
 730
 ----
 8766 hours in one year.
   20
 ------
 175320 hours in 20 years.
     60
 --------
 10519200 minutes in do.
       60
 ---------
 631152000 ſeconds in do.
```

```
      Proof.
6|0)63115200|0
  -----------
6|0)1051920|0
  ----------
2|0)17532|0
  --------
4×6)8766
   -----
  4)1461
   -----
   365d. 6h.
```

2. Suppoſe your age to be 15y. 19d. 11h. 37m. 45s. how many ſeconds are there in it, allowing 365 days 6 hours to the year? Ans. 475047465.

3. How many minutes from the firſt day of January to the 14th day of Auguſt, incluſively?

Ans. 325440.

4. How many days ſince the commencement of the Chriſtian era?

5. How many minutes ſince the commencement of the American war, which happened on the 19th day of April, 1775?

6. How many ſeconds between the commencement of the war, April 19th, 1775, and the Independence of the United States of America, which took place the 4th day of July, 1776? Ans. 38102400.

8. LAND, OR SQUARE MEASURE.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

Acr. r. p.
29 3 19
4
——
119 roods.
40
——
Ans. 4779 perches.

Proof.
4|0)477|9
——
4)119 19p.
——
29a. 3r.

2. In 1997 poles, how many acres?
Ans. 12a. 1r. 37p.

3. In 89763 ſquare yards, how many acres, &c.?
Ans. 18a. 2r. 7p. 101ft. 36in.

9. SOLID MEASURE.

1. In 15 tons of hewn timber, how many ſolid inches?

15 tons.
50
——
750 feet.
1728
——
6000
1500
5250
750
——
1296000 inches, Ans.

Proof.
5|0
1728)1296000(75|0
12096
——
8640
8640
——
15 To.

2. In 9 tons of round timber, how many inches?
Ans. 622080.

3. In 25 cords of wood, how many inches?
Ans. 5529600.

10. WINE MEASURE.

1. In 9hhd. 15gal. 3qt. of wine, how many quarts?

hhd. gal. qt.
9 15 3
63
32
55
582 gallons.
4
2331 quarts, Ans.

Proof.
4)2331
63)582 3qts.
9hhd. 15gal.

2. In 12 pipes of wine, how many pints?
Ans. 12096.

3. In 9758 pints of brandy, how many pipes?
Ans. 9p. 1hhd. 22gal. 3qts.

11. ALE AND BEER MEASURE.

1. In 29hhd. of beer, how many pints?

hhd.
29
54
116
145
1566 gallons.
4
6264 quarts.
2
12528 pints.

Proof.
2)12528
4)6264
54)1566
29hhd.

2. In 47 barrels, 18gal. of ale, how many pints?
Ans. 13680.

3. In 36 puncheons of beer, how many butts?
Ans. 24.

12. Dry Measure.

1. In 42 chaldrons of coals, how many pecks?

Chaldrons.	
42	Proof.
32	4)5376
84	32)1344(42
126	128
1344 buſhels.	64
4	64
Ans. 5376 pecks.	

2. In 75 buſhels of corn, how many pints? Ans. 4800.

3. In 9376 quarts, how many buſhels? Ans. 293.

VULGAR FRACTIONS.

FRACTIONS, or broken numbers, are expreſſions for any aſſignable parts of a unit, or whole number; and are repreſented by two numbers, placed one above another, with a line drawn between them, thus, $\frac{5}{8}$, $\frac{4}{3}$, &c. ſignifying five eighths, four thirds, that is, one and one third, &c.

The figure above the line is called the numerator, and that below, the denominator.

The denominator (which is the diviſor in diviſion) ſhows how many parts the integer is divided into; and the numerator (which is the remainder after diviſion) ſhows how many of thoſe parts are meant by the fraction.

Fractions are either proper, improper, ſingle, compound, or mixed. Any whole number may be made an improper fraction, by drawing a line under it, and putting unity, or 1, for a denominator, as 9 may be expreſſed fractionwiſe, thus, $\frac{9}{1}$, and 12 thus, $\frac{12}{1}$, &c.

1. A *ſingle* or *ſimple* fraction is a fraction expreſſed in a ſimple form; as, $\frac{1}{2}$, $\frac{5}{9}$, $\frac{7}{12}$ &c.

2. A *compound* fraction is a fraction expreſſed in a compound form, being a fraction of a fraction; or two or more fractions connected together; as, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{5}{11}$ of $\frac{19}{20}$; which are read thus, one half of three fourths, two ſevenths of five elevenths of nineteen twentieths, &c.

3. A *proper* fraction is a fraction, whoſe numerator is leſs than its denominator; as, $\frac{2}{5}$, $\frac{3}{8}$, &c.

4. An *improper* fraction is a fraction, whoſe numerator exceeds its denominator; as, $\frac{5}{2}$, $\frac{8}{5}$, &c.

5. A *mixed number* is compoſed of a whole number and a fraction; as, $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, ſeven and three fifths, &c.

6. A fraction is ſaid to be in its *leaſt* or *loweſt terms*, when it is expreſſed by the leaſt numbers poſſible.

7. The *common meaſure* of two, or more numbers, is that number, which will divide each of them without a remainder; Thus, 5 is the common meaſure of 10, 20, and 30; and the *greateſt* number, which will do this, is called the *greateſt common meaſure*.

8. A number, which can be meaſured by two, or more, numbers, is called their *common multiple*: And, if it be the leaſt number, which can be ſo meaſured, it is called the *leaſt common multiple*; thus, 40, 60, 80, 100, are multiples of 4 and 5; but their leaſt common multiple is 20.

9. A *prime number* is that, which can only be meaſured by itſelf, or an unit.

10. That number, which is produced by multiplying ſeveral numbers together, is called a *compoſite number*.

11. A *perfect number* is equal to the ſum of all its aliquot parts.

PROBLEM I.

To find the greateſt common meaſure of two or more numbers.

RULE.

1. If there be two numbers only, divide the greater by the leſs, and this diviſor by the remainder, and ſo on, always dividing the laſt diviſor by the laſt remainder, till nothing remain, then will the laſt diviſor be the greateſt common meaſure required.

2. When there are more than two numbers, find the greateſt common meaſure of two of them, as before;

then of that common meaſure and one of the other numbers, and ſo on, through all the numbers to the laſt; then will the greateſt common meaſure, laſt found, be the anſwer.

3. If 1 happens to be the common meaſure, the given numbers are prime to each other, and found to be incommenſurable, or in their loweſt terms.

EXAMPLES.

1. What is the greateſt common meaſure of 1836, 3996, and 1044?

1836)3996(2
3672
324)1836(5
1620
216)324(1
216
Com. meaſ.=108)216(2
216

So 108 is the greateſt common meaſure of 3996 & 1836;
Hence 108)1044(9
972
72)108(1
72
Laſt gr. com. meaſ.=36)72(2
72

Therefore 36 is the anſwer required.

2. What is the greateſt common meaſure of 1224 and 1080? Ans. 72.

PROBLEM II.

To find the leaſt common multiple of two, or more, numbers.

1. Divide by any number, that will divide two or more of the given numbers, without a remainder, and ſet the quotients, together with the undivided numbers, in a line beneath.

2. Divide the ſecond line as before; and ſo on, till there are no two numbers, that can be divided; then, the continued product of the diviſors and quotients will give the multiple required.

EXAMPLES.

1. What is the leaſt common multiple of 6, 10, 16, and 20?

*5)	6	10	16	20
*2)	6	2	16	4
*2)	3	1	8	2
	*3	1	*4	1

$\overset{*}{5}\times\overset{*}{2}\times\overset{*}{2}\times\overset{*}{3}\times\overset{*}{4}=240$ Ans.

I ſurvey my given numbers, and find that 5 will divide two of them, viz. 10 and 20, which I divide by 5, bringing, into a line with the quotients, the numbers, which 5 will not meaſure: again, I view the numbers in the ſecond line, and find 2 will meaſure them all, and I get 3, 1, 8, 2, in the third line, and find that 2 will meaſure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime; I then multiply the prime numbers and the diviſors continually into each other, for the number ſought, and find it to be 240.

2. What is the leaſt common multiple of 6 and 8? Ans. 24.

3. What is the leaſt number that 3, 5, 8, and 10 will meaſure? Ans. 120.

4. What is the leaſt number, which can be divided by the 9 digits ſeparately, without a remainder? Ans. 2520.

REDUCTION OF VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.

To abbreviate, or reduce fractions to their loweſt terms.

RULE.*

Divide the terms of the given fraction by any number, which will divide them without a remainder, and the quotients again in the ſame manner; and ſo on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its loweſt terms.

---

* That dividing both the terms, that is, both numerator and denominator of the fractions, equally by any number, whatever, will give another fraction, equal to the former, evident : And if thoſe

Or, Divide both the terms of the fraction by their greateſt common meaſure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its loweſt terms.

$$8\Big\}\ \frac{288}{480}=\overset{(4)}{\frac{36}{60}}=\overset{(3)}{\frac{9}{15}}=\frac{3}{5}\ \text{the answer.}$$

Or thus:

288)480(1
288
———
192)288(1
192
———
Com. mea. 96)192(2
192
———

Therefore 96 is the greateſt common meaſure;

and $96\Big\}\ \frac{288}{480}=\frac{3}{5}$ the ſame as before.

2. Reduce $\frac{57}{456}$ to its loweſt terms. Ans. $\frac{1}{8}$.

3. Reduce $\frac{46}{184}$ to its loweſt terms. Ans. $\frac{1}{4}$.

4. Reduce $\frac{1429}{2858}$ to its loweſt terms. Ans. $\frac{1}{2}$.

---

diviſions be performed as often as can be done, or the common diviſor be the greateſt poſſible, the terms of the reſulting fraction muſt be the leaſt poſſible.

NOTE 1. When numbers with the ſign of Addition or Subtraction between them are to be divided by any numbers, each of the numbers muſt be divided: Thus, $\frac{6+9+12}{3}=2+3+4=9$.

2. But if the numbers have the ſign of Multiplication between them; then only one of them muſt be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which ſubjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ \times 8+5 \\ \hline \text{Ans. } 293 \\ \hline 8 \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the ſum, 293, is the numerator of the fraction ſought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{\overline{36\times8+5}}{8}=\frac{293}{8}$. Ans. as before.

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction.
Ans. $\frac{2163}{17}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$.

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the ſaid denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whoſe denominator ſhall be 8. $6\times8=48$, and $\frac{48}{8}$ the Ans.—Proof. $\frac{48}{8}=48\div8=6$.

2. Reduce 15 to a fraction, whoſe denominator ſhall be 12.
Ans. $\frac{180}{12}$.

---

* All fractions repreſent a diviſion of a numerator by the denominator, and are taken altogether as proper and adequate expreſſions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

CASE IV.*

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole or mixed number.

$8)293(36\frac{5}{8}$ Ans.

24

53

48

5

Or, $\frac{293}{8}=293\div8=36\frac{5}{8}$ as before.

2. Reduce $\frac{2163}{17}$ to its equivalent whole or mixed number. Ans. $127\frac{4}{17}$.

3. Reduce $\frac{45}{5}$ to its equivalent whole number. Ans. 9.

CASE V.

To reduce a compound fraction to an equivalent ſimple one.

RULE.

Multiply all the numerators continually together for a new numerator, and all the denominators for a new denominator, and they will form the ſimple fraction required.

If part of the compound fraction be a whole or mixed number, it muſt be reduced to an improper fraction, by caſe 2d or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, theſe equal numerators and denominators may be expunged, and the other members continually multiplied, (as by the rule) will produce the fractions required in lower terms.

---

* This caſe is evidently the reverſe of Caſe 2d.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a ſimple fraction.

$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5}$ the Ans.

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a ſimple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ Ans. Or, by expunging the equal numerators and denominators, it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ as before.

3. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a ſimple fraction.
Ans. $\frac{75}{64}=1\frac{11}{64}$.

CASE VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE I.

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other continually, for a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ the new numerator for $\frac{1}{4}$.
$2\times4\times8=64$ ditto $\frac{2}{5}$.
$5\times4\times5=100$ ditto $\frac{5}{8}$.
$4\times5\times8=160$ the common denominator.

Therefore the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, and $\frac{100}{160}$, the anſwer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$, to fractions, having a common denominator.
Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$, and $\frac{3}{13}$ to a common denominator. Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

RULE II.

To reduce any given fractions to others, which ſhall have the leaſt common denominator.

1. By Problem 2, (page 56) find the leaſt common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ to fractions having the leaſt common denominator poſſible.

$$\begin{array}{r|lll} 4 & 3 & 4 & 8 \\ \hline & 3 & 1 & 2 \\ \hline \end{array}$$

$4 \times 3 \times 2 = 24 =$ leaſt common denominator.

$\overline{24 \div 3} \times 1 = 8$ the firſt numerator ; $\overline{24 \div 4} \times 3 = 18$ the ſecond numerator ; $\overline{24 \div 8} \times 7 = 21$ the third numerator.

Whence the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the leaſt common denominator. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the ſame value.

RULE.

Reduce the given fraction to a compound one by comparing it with all the denominations between it and that denomination you would reduce it to ; laſtly, reduce this compound fraction to a ſingle one, by Caſe V. and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

1. Reduce $\frac{4}{7}$ of a cent to the fraction of a dollar. By comparing it, it becomes $\frac{4}{7}$ of $\frac{1}{10}$ of $\frac{1}{10}$, which reduced by Caſe V. will be $\frac{4\times1\times1}{\text{and }7\times10\times10}=\frac{4}{700}=\frac{1}{175}$ Dol. Ans.

2. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound. Ans £$\frac{1}{400}$.

3. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb. Avoirdupois. Ans. $\frac{1}{28}$lb.

4. Reduce $\frac{7}{8}$ of a pennyweight to the fraction of a lb. Troy. Ans. $\frac{7}{1920}$lb.

5. Reduce $\frac{8}{9}$ of a lb. Avoirdupois to the fraction of a Cwt. Ans. $\frac{1}{126}$Cwt.

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but leſs, retaining the ſame value.

RULE.

Multiply the given numerator by the parts of the denominations between it and that denomination you would reduce it to, for a new numerator, which place over the given denominator : Or, only invert the parts contained in the integer, and make of them a compound fraction as before ; then, reduce it to a ſimple one.

EXAMPLES.

1. Reduce $\frac{1}{175}$ of a dollar to the fraction of a cent. By comparing the fraction it will be $\frac{1}{175}$ of $\frac{10}{1}$ of $\frac{10}{1}$; then $\frac{1\times10\times10}{175\times1\times1}=\frac{100}{175}=\frac{4}{7}$c. Ans.

2. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny. Ans $\frac{3}{5}$d.

3. Reduce $\frac{1}{28}$ of a lb. Avoirdupois to the fraction of an ounce. Ans. $\frac{4}{7}$oz.

4. Reduce $\frac{7}{1920}$ of a lb. Troy to the fraction of a pwt. Ans. $\frac{7}{8}$pwt.

5. Reduce $\frac{1}{120}$ of a cwt. to the fraction of a lb. Avoirdupois. Ans. $\frac{8}{9}$lb.

CASE IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, meaſure, &c.

Rule.

Multiply the numerator by the parts in the next inferiour denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferiour denomination, and divide by the denominator as before, and ſo on, as far as neceſſary; and the quotients placed after one another, in their order, will be the anſwer required. Or, Reduce the numerator, as if it were a whole number, to the loweſt denomination, and divide the reſult by the denominator; the quotient will be the number of the loweſt denomination, (which muſt be brought into higher denominations as far as it will go) and the remainder will be a numerator to be placed over the given denominator for a fraction of the loweſt denomination.

Note. From this rule, in connection with what has been ſaid of Reduction of Federal Money, it appears, that, annexing to the given numerator as many cyphers, as will fill all the places to the loweſt denomination, and dividing the number ſo formed by the denominator, the quotient will be the anſwer in the ſeveral denominations, and the remainder a numerator to be placed over the given denominator, forming a fraction of the loweſt denomination.

Examples.

1. What is the value of $\frac{5}{8}$ of a dollar?

By the general rule.

```
      5
     10
   8)50(  2
     --
   d. 6   10
          --
      8)20(  4
        --
      c. 2   10
             --
          8)40
            --
         m. 5
```

By the note.

```
   D. d. c. m.
 8)5  0  0  0
   ----------
      6  2  5
```

Ans. 6d. 2c. 5m.
or 62c. 5m.

Or thus.

5D.=5000m. and $\frac{5000}{8}$m.=625m.=62c. 5m. anſwer, as before.

2. What is the value of $\frac{17}{64}$ of a dollar ?

```
     D.  d.  c.  m.
 64)17   0   0   0
    128
    ---
     420
     384
     ---
      360
      320
      ---
       40
       -- = 5/8
       64
```

(2d. 6c. $5\frac{5}{8}$m.

or 26c. $5\frac{5}{8}$m.

Or, 17D.=17000m. And $\frac{17000}{64}$m.=$265\frac{5}{8}$m.= 26c. $5\frac{5}{8}$m. Ans. as before.

3. What is the value of $\frac{5}{7}$ of a pound ?

Ans 14s. 3d. $1\frac{5}{7}$qr.

4. What is the value of $\frac{4}{5}$ of a lb. Avoirdupois ?

Ans. 12oz. $12\frac{4}{5}$dr.

5. What is the value of $\frac{5}{6}$ of a mile ?

Ans. 6fur. 26pol. 11ft.

6. What is the value of $\frac{9}{13}$ of a day ?

Ans. 16h. 36m. $55\frac{5}{13}$s.

7. What is the value of $\frac{6}{7}$ of an acre ?

Ans. 3roods $17\frac{1}{7}$p.

CASE X.*

To reduce any given quantity to the fraction of any greater denomination of the ſame kind.

Rule.

Reduce the given quantity to the loweſt term mentioned, for a numerator; then reduce the integral part to the ſame term for a denominator; which will be the fraction required.

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* This caſe is the reverſe of the former, therefore proves it.

Note. If there be a fraction given with the ſaid quantity, it muſt be farther reduced to the denominative parts thereof, adding thereto the numerator.

Note. It appears from this rule, and what has been ſaid before, that, in Federal Money, where the given quantity contains no fraction of its loweſt denomination, the annexing of as many cyphers to 1 of the required denomination, as will extend to the loweſt denomination in the given quantity, will form a denominator, which placed under the given quantity uſed as one number for a numerator, will make the anſwer, which may be reduced to its loweſt terms. Or, if there be a fraction of the loweſt denomination, multiply the given whole numbers by its denominator, adding its numerator, for a numerator; and let the denominator itſelf, at the left of as many cyphers as were mentioned above, be a denominator; the fraction ſo formed will be the anſwer; which may be reduced to its loweſt terms.

Examples.

1. Reduce 6d. 2c. 5m. to the fraction of a dollar.

By the general rule.

6d.	10d. int. part.	
×10+2	10	
62	100	By the note.
×10+5	10	D. d. c. m.
625	1000	$\frac{6\ \ 2\ \ 5}{1\ \ 0\ \ 0\ \ 0}=\frac{5}{8}$D.
And $\frac{625}{1000}=\frac{5}{8}$D. Ans.		Ans. as before.

2. Reduce 26c. $5\frac{5}{8}$m. to the fraction of a dollar.

By the general rule.

26c.	100c. int. pt.
×10+5	10
265	1000
×8+5	8
2125	8000

And $\frac{2125}{8000}=\frac{17}{64}$D. Ans.

By the note.

—— D. d. c. m.
265×8+5+2 1 2 5
———————— $=\frac{17}{64}$D.
And 1D×8=8 0 0 0

Ans. as before.

3. Reduce 14s. $3\frac{1}{4}$d. $\frac{6}{7}$ to the fraction of a pound.
Ans. $\frac{4800}{6720}=\frac{5}{7}$£.

4. Reduce 12oz. $12\frac{4}{5}$dr. to the fraction of a lb. Avoirdupois.
Ans. $\frac{4}{5}$lb.

5. Reduce 6fur. 26po. 11ft. to the fraction of a mile.
Ans. $\frac{5}{8}$m.

6. Reduce 16h. 36m. $55\frac{5}{13}$s. to the fraction of a day.
Ans. $\frac{9}{13}$ day.

7. Reduce 3r. $17\frac{1}{7}$po. to the fraction of an acre.
Ans. $\frac{6}{7}$ acre.

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to ſingle ones; mixed numbers to improper fractions; fractions of different integers to thoſe of the ſame; and all of them to a common denominator; then, the ſum of the numerators written over the common denominator will be the ſum of the fractions required.

EXAMPLES.

1. Add $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$, and 7 together.

Firſt, $7\frac{4}{5}=\frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8}=\frac{15}{56}$, and $7=\frac{7}{1}$.
Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; therefore,

$$\begin{array}{r} 39\times56\times\ 1=2184 \\ 15\times\ 5\times\ 1=\ \ 75 \\ 7\times\ 5\times56=1960 \\ \hline \end{array}$$

$$\frac{4219}{5\times56\times1=280}=15\frac{19}{280}.$$

Or thus, $\frac{2184+75+1960}{280}=15\frac{19}{280}$.

2. What is the ſum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$?
Ans. $12\frac{59}{80}$.

3. Add together $\frac{1}{3}$D. $\frac{5}{8}$C. $\frac{3}{10}$c. and $\frac{7}{8}$m.
Ans. 20c. 9m.

4. Add $\frac{1}{9}$l. $\frac{3}{7}$s. and $\frac{4}{3}$d. together. Ans. 2s. $8\frac{64}{105}$d.

5. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{8}$ of an hour, and $\frac{3}{4}$ of a minute together.
Ans. 2 days, 2 hours, 30 minutes, 45 ſeconds.

SUBTRACTION OF VULGAR FRACTIONS.

Rule.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

Examples.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8}=\frac{10}{56}=\frac{5}{28}$. Then the fractions are $\frac{3}{4}$ and $\frac{5}{28}$.

$$\left.\begin{array}{l}3\times28=84\\5\times4=20\\4\times28=112\text{ com. den.}\end{array}\right\}\begin{array}{l}\frac{3}{4}=\frac{84}{112},\text{ and }\frac{5}{28}=\frac{29}{112},\text{ therefore,}\\\frac{84}{112}-\frac{20}{112}=\frac{64}{112}=\frac{4}{7}\text{ remainder.}\end{array}$$

2. From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.

---

* In ſubtracting mixed numbers, when the fractions have a common denominator, and the numerator in the ſubtrahend is leſs than that in the minuend, the difference of the whole numbers will be a whole number, and the difference of the numerators a numerator to be placed over the given denominator: this whole number and the fraction thus formed will be the remainder: but, when the numerator in the ſubtrahend is greater than that in the minuend, ſubtract the numerator in the ſubtrahend from the common denominator, adding the numerator in the minuend, and carrying one to the integer of the ſubtrahend.

Hence, A fraction is ſubtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

Examples.

From	$12\frac{3}{5}$	$12\frac{2}{5}$	12
Take	$7\frac{2}{5}$	$7\frac{3}{5}$	$\frac{2}{5}$
Rem.	$5\frac{1}{5}$	$4\frac{4}{5}$	$11\frac{3}{5}$

3. Take $3\frac{1}{3}$c. from $\frac{1}{5}$ of $2\frac{1}{3}$D. Ans. $43\frac{1}{3}$c.

4. From $\frac{1}{4}$l. take $\frac{9}{10}$s. Ans. 4s. $1\frac{1}{5}$d.

5. From 5 weeks take $19\frac{4}{5}$ days.
Ans. 15da. 4ho. 48min.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.

NOTE. Where ſeveral fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\frac{1}{3}=\frac{13}{3},\ \frac{1}{4} \text{ of } \frac{7}{8}=\frac{1\times7}{4\times8}=\frac{7}{32}, \text{ and } 6=\frac{6}{1}.$$

$$\text{Then } \frac{13}{3}\times\frac{1}{5}\times\frac{7}{32}\times\frac{6}{1}=\frac{13\times1\times7\times6}{3\times5\times32\times1}=\frac{546}{480}=1\frac{11}{80} \text{ the anſwer.}$$

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{20}{459}$.
3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{7}{8}$.
4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. Ans. $\frac{5}{14}$.

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before; then, invert the diviſor and proceed exactly as in Multiplication: The products will be the quotient required.

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* To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

EXAMPLES.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$\frac{1}{3}$ of $17=\frac{1}{3}$ of $\frac{17}{1}=\frac{1\times17}{3\times1}=\frac{17}{3}$, and $\frac{2}{3}$ of $\frac{6}{8}=\frac{12}{24}=\frac{1}{2}$; therefore, $\frac{17}{3}\div\frac{1}{2}=\frac{17\times2}{3\times1}=\frac{34}{3}=11\frac{1}{3}$ the quotient required.

2. Divide $\frac{6}{7}$ by $\frac{3}{5}$. Ans. $1\frac{4}{21}$.
3. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.
4. Divide $\frac{3}{7}$ by 9. Ans. $\frac{1}{21}$.
5. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$. Ans. $\frac{8}{9}$.

DECIMAL FRACTIONS.

DECIMAL Fractions are of ſuch a nature, that they vary in the ſame proportion, and are managed by the ſame method of operation, as whole numbers are.

On this account, every proper fraction is ſuppoſed to be reducible to another, whoſe denominator ſhall be 10, 100, 1000, &c. viz. Unity, with a number of cyphers annexed; and fractions with ſuch denominators are called *Decimal Fractions*: Such are $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, 100, 1000, &c. the denominators need not be expreſſed: For the numerator only may be made to expreſs the true value: For this purpoſe it is only required to write the numerator with a point before it at the left hand, to diſtinguiſh it from a whole number, when it conſiſts of ſo many figures, as the denominator hath cyphers annexed to unity, or 1: So $\frac{5}{10}$ is written ·5; $\frac{33}{100}$, ·33; $\frac{735}{1000}$, ·735, &c.

NOTE. The point prefixed is called a Separatrix.

But if the numerator has not ſo many places as the denominator has cyphers, put ſo many cyphers before it, viz. at the left hand, as will make up the defect: So write $\frac{5}{100}$ thus. ·05; and $\frac{6}{1000}$ thus, ·006, &c. And thus do theſe fractions receive the form of whole numbers.

The 1ſt, 2d, 3d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, ſeconds, thirds, fourths, &c.

We may conſider unity as a fixed point, from whence whole numbers proceed infinitely increaſing toward the left hand, and decimals infinitely decreaſing toward the right hand to 0, as in the following

TABLE.*

C Millions	X Millions	Millions	C Thouſands	X Thouſands	Thouſands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thouſandth Parts	X Thouſandth Parts	C Thouſandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1·	2	3	4	5	6	7	8	9

---

* It will be very apparent to the learner from the nature of decimals, and what has been ſaid of Federal Money, that this money is purely decimal ; and the dollar being the money-unit, the lower denominations are plainly ſo many decimal parts of a dollar ; thus 9 dollars and 8 dimes are expreſſed $9{\cdot}8=9\frac{8}{10}$ doll.—12 dollars, 4 dimes and 7 cents thus, $12{\cdot}47=12\frac{47}{100}$ doll.—20 dollars, 3 dimes, 4 cents, and 5 mills, thus, $20{\cdot}345=20\frac{345}{1000}$ doll.—100 dollars and 9 mills, thus, $100{\cdot}009=100\frac{9}{1000}$ doll.—50 dollars and 5 cents, thus, $50{\cdot}05=50\frac{5}{100}$ doll. ; wherefore, it is, in all reſpects, added, ſubtracted, multiplied, and divided, the ſame as decimals ; and, of all coins, it is the moſt ſimple.

It may alſo be obſerved, that the ſum exhibits the particular number of each different piece of money contained in it, viz. 455997 mills$=45599\frac{7}{10}$ cents$=4559\frac{97}{100}$ dimes$=455\frac{997}{1000}$ dollars$=$

E. D. d. c. m.

$45\frac{5997}{10000}$ eagles$=4\ 5\ 5{\cdot}\ 9\ 9\ 7$.

Alſo, the names of the coins, leſs than a dollar, are ſignificant of their values. For the *mill*, which ſtands in the third place at the right hand of the ſeparatrix, or place of thouſandths, is contracted from *mille*, the Latin for *thouſand* : *Cent*, which occupies the ſecond place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred* : And *dime*, which is in the firſt place, is derived from *diſme*, the French for *tenths*.

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its diſtance from unit's place: If it be in the firſt place after units (or the ſeparating point) it ſignifies tenths; if in the ſecond, hundreds, &c. decreaſing in each place in a tenfold proportion.

Conſequently, every ſingle figure expreſſing a decimal has for its denominator an unit, or 1, with ſo many cyphers as its place is diſtant from unit's place: Thus 2 in the decimal part of the table=$\frac{2}{10}$; 3=$\frac{3}{100}$; 4=$\frac{4}{1000}$, &c. And if a decimal be expreſſed by ſeveral figures, the denominator is 1, with ſo many cyphers as the loweſt figure is diſtant from unit's place. So ·357 ſignifies $\frac{357}{1000}$, and ·0053=$\frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to poſſeſs the ſame place: So ·5, ·50, and ·500, are all of the ſame value, and each equal to $\frac{1}{2}$.

But cyphers placed at the left hand of a decimal, do alter its value, every cypher depreſſing it to $\frac{1}{10}$ of the value it had before, by removing every ſignificant figure one place further from the place of units. So ·5, ·05, ·005, all expreſs different decimals, viz. ·5, $\frac{5}{10}$; ·05, $\frac{5}{100}$; ·005, $\frac{5}{1000}$.

Hence may be obſerved the contrary effects of cyphers being annexed to whole numbers and decimals.

It is likewiſe evident from the table, that ſince the places of decimals decreaſe in a tenfold proportion from units downwards, ſo they conſequently increaſe in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100}=\frac{1}{10}$, $\frac{10}{10}=1$, and $1\times10=10$, which proves that decimals are ſubject to the ſame law of Notation, and conſequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed

Such being the nature of Federal Money, its operations can in no other way be ſo well underſtood, as in obtaining a good knowledge of decimals, and applying their ſeveral rules to the various caſes of money matters.

to the right hand of thoſe, which have fewer places, ſo many cyphers as make them equal in places with that which has the moſt. So theſe decimals, ·5, ·06, ·455, may be reduced to the decimals ·500, ·060, ·455, which have all 1000 for their denominator.

Of decimals, that is the greateſt, whoſe higheſt figure is greateſt, whether they conſiſt of an equal or unequal number of places: Thus, ·5 is greater than ·459, for if it be reduced to the ſame denominator with ·459, it will be ·500.

A mixed number, viz. a whole number with a decimal annexed, is equal to an improper fraction, whoſe numerator is all the figures of the mixed number, taken as one whole number, and the denominator that of the decimal part So 45·309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction: Thus, 45×1000+·309=$\frac{45309}{1000}$ as above.

ADDITION OF DECIMALS.

RULE.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their ſum, as in whole numbers, and point off ſo many places for decimals, as are equal to the greateſt number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the ſum of 19·073+2·3597+223+·0197581+3478·1+12·358.

```
  19·073
   2·3597
 223·
    ·0197581
3478·1
  12·358
------------
3734·9104581 the ſum.
```

2. Required the ſum of 429+21·37+355·003+1·07+1·7 ? Ans. 808·143.

3 Required the ſum of 973+19+1·75+93·7164+·9501 ? Ans. 1088·4165.

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value: Then ſubtract as in whole numbers, and point off the decimals as in Addition.

EXAMPLES.

1. Find the difference of 1793·13 and 817·05693.

From 1793·13
Take 817·05693

Remainder 976·07307

2. From 171·195 take 125·9176. Ans. 45·2774.
3. From 219·1384 take 195·91. Ans. 23·2284.
4. From 480 take 245·0075. Ans. 234·9925.

MULTIPLICATION OF DECIMALS.

RULE

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off ſo many figures from the product as there are decimal places in both the factors; and if there be not ſo many places in the product, ſupply the defect by prefixing cyphers.

EXAMPLES.

1. Multiply ·02345
by ·00163

7035
14070
2345

·0000382235 the product.

2. Multiply 25·238 by 12·17. Ans. 307·14646.
3. Multiply ·3759 by ·945. Ans. ·3552255.
4. Multiply ·84179 by ·0385. Ans. ·032408915.

To multiply by 10, 100, 1000, &c. remove the ſeparating point ſo many places to the right hand, as the multiplier has cyphers.

So ·345 multiplied by $\left\{\begin{array}{l}10\\100\\1000\end{array}\right\}$ makes $\left\{\begin{array}{l}3{\cdot}45\\34{\cdot}5\\345\end{array}\right.$

For ·345×10 is 3·450, &c.

DIVISION OF DECIMALS.

Rule.

1. The places of decimal parts in the diviſor and quotient, counted together, muſt always be equal to thoſe in the dividend; therefore, divide as in whole numbers, and, from the right hand of the quotient point off ſo many places for decimals, as the decimal places in the dividend exceed thoſe in the diviſor.

2. If the places of the quotient be not ſo many as the rule requires, ſupply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the diviſor be more than thoſe in the dividend, cyphers may be annexed to the dividend or to the remainder, and the quotient carried on to any degree of exactneſs.

Examples.

1.

```
219)·117841075(·0005380875 &c.
     1095
     ----
      834
      657
      ----
      1771
      1752
      ----
       1907
       1752
       ----
        1555
        1533
        ----
          22
```

2.

·3719)38·0000(102·178, &c.
3719
8100
7438
6620
3719
29010
26033
29770
29752
18

In Example 1ſt, the diviſor having no decimals, the quotient muſt have ſo many as there are in the dividend. In Example 2d, the dividend being an integer, muſt have at leaſt ſo many cyphers annexed, as there are decimals in the diviſor, and ſo far the quotient will be whole numbers; then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the rule.

3. 133)5737(43·1353+
4. 23·7)65321(2756·16+
5. *72)918·217(12753+
6. 25·17)315·6293(1253+
7. ·317)29·417(92+

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers) it is performed by removing the ſeparatrix, in the dividend, ſo many places toward the left hand, as there are cyphers in the diviſor.

$$\text{So} \left\{\begin{array}{r}10\\100\\1000\\10000\end{array}\right\} \text{dividing 7654, the quotient is} \left\{\begin{array}{l}765{\cdot}4\\76{\cdot}54\\7{\cdot}654\\{\cdot}7654\end{array}\right.$$

---

* The following queſtions are left unpointed in the quotient, to exerciſe the learner.

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal.

RULE.

Divide the numerator by the denominator, as in Diviſion of Decimals, and the quotient will be the decimal required.—Or, So many cyphers as you annex to the given numerator, ſo many places muſt be pointed off in the quotient, and if there be not ſo many places of figures in the quotient, the deficiency muſt be ſupplied by prefixing ſo many cyphers before the quotient figures.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal. 8)1·000

·125 Ans.

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{2}{3}$ to decimals.
Ans. ·375, ·625, ·666+.

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to decimals.
Ans. ·25, ·5, ·75, ·333+, ·8, ·833+, ·875.

4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$, and $\frac{9}{36}$ to decimals.
Ans. ·263+, ·692+, ·025, ·25.

CASE II.

To reduce numbers of different denominations, as of Money, Weight, and Meaſure, to their equivalent decimal values.

RULE.

1. Write the given numbers perpendicularly under each other, for dividends, proceeding orderly from the leaſt to the greateſt.

2. Oppoſite to each dividend, on the left hand, place ſuch a number, for a diviſor, as will bring it to the next ſuperiour denomination, and draw a line perpendicularly between them.

3. Begin with the higheſt, and write the quotient of each diviſion as decimal parts on the right hand of the dividend next below it, and ſo on, until they are all uſed, and the laſt quotient will be the decimal ſought.

Examples.

1. Reduce 17s. 8¾d. to the decimal of a pound.

4 | 3·
12 | 8·75
20 | 17·729166, &c.

·886458, &c. the decimal required.

Here, in dividing 3 by 4, I ſuppoſe two cyphers to be annexed to the 3, which make it 3·00, and ·75 is the quotient, which I write againſt 8 in the next line; this quotient, viz. 8·75, being pence and decimal parts of a penny, I divide them by 12, which brings them to ſhillings and decimal parts; I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce 1, 2, 3, 4, and ſo on to 19 ſhillings, to decimals.

Shillings.	1	2	3	4	5	6
Anſwers.	·05,	·1,	·15,	·2	·25,	·3,

Shillings.	7	8	9	10	11	12
Anſwers.	·35,	·4,	·45,	·5,	·55,	·6,

Shillings.	13	14	15	16	17	18	19
Ans.	·65,	·7,	·75,	·8,	·85,	·9,	·95.

Here, when the ſhillings are even, half the number, with a point prefixed, is their decimal expreſſion; but if the number be odd, annex a cypher to the ſhillings, and then halving them, you will have their decimal expreſſion.

3. *Reduce 1, 2, 3, and ſo on to 11 pence, to the decimals of a ſhilling.

Pence.	1	2	3	4	5	6
Anſwers.	·083+,	·166,	·25,	·333+,	·416+,	·5,

Pence.	7	8	9	10	11
Anſwers.	·583+,	·666+,	·75,	·833+,	·916+.

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

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* The anſwers to this queſtion are the ſame as the decimal parts of a foot.

Pence.	1	2	3	4
Anſwers.	·00416+,	·0083+,	·0125,	·01666+.
Pence.	5	6	7	8
Anſwers.	·0208+,	·025,	·02916+,	·0333+,
Pence.	9	10.	11.	
Anſwers.	·0375,	·0416+,	·04583+.	

5. Reduce 1, 2, and 3 farthings to the decimals of a penny. Ans. 1qr.=·25d. 2qrs.=·5d. and 3qrs.=·75d.

6. Reduce 1, 2, and 3 farthings to the decimals of a ſhilling.
Ans. 1qr.=·02083+s. 2qrs.=·04166+s. 3qrs.=·0625s.

7. Reduce 1, 2, and 3 farthings to the decimals of a pound.
Ans. 1qr.=£·0010416+ 2qrs.=£.002083+ 3qrs.=£·003125.

8. Reduce 13s. 5½d. to the decimal of a pound.
Ans. ·6729+.

9. Reduce 7cwt. 3qrs. 17lb. 10oz. 12dr. to the decimal of a ton. Ans. ·39538+.

10. Reduce 10oz. 13pwt. 9gr. to the decimal of a pound Troy. Ans. ·8890625.

11. Reduce 3qrs. 3n. to the decimal of a yard.
Ans. ·9375.

CASE III.

To find the decimal of any number of ſhillings, pence, and farthings, by inſpection.

Rule.

1. Write half the greateſt even number of ſhillings for the firſt decimal figure.

2. Let the farthings in the given pence and farthings poſſeſs the ſecond and third places; obſerving to increaſe the ſecond place, or place of hundredths, by 5, if the ſhillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

Examples.

1. Find the decimal of 13s. 9¾d. by inſpection.

·6 . . =½ of 12s.
5 for the odd ſhilling.
39 = farthings in 9¾d.
Add 2 for the exceſs of 36.

·691 = decimal required.

2 Find by inſpection the decimal expreſſions of 18s. 3¼d. and 17s. 8½d. Ans. £·914 and £·885.

CASE IV.

To find the value of any given decimal in the terms of the integer.

Rule.—1. Multiply the decimal by the number of parts in the next leſs denomination, and cut off ſo many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferiour denomination, and cut off a remainder as before.

3 Proceed in this manner through all the parts of the integer, and the ſeveral denominations, ſtanding on the left hand, make the anſwer.

Examples.

1. Find the value of ·73968 of a pound.

```
    ·73968
        20
  --------
  14·79360
        12
  --------
   9·52320
         4
  --------
   2·09280      Ans. 14s. 9½d.
```

2. What is the value of ·679 of a ſhilling? Ans. 8·148d.

3. What is the value of ·617 of a cwt.? Ans. 2qrs. 13lb. 1oz. 10$\frac{6}{10}$dr.

4. What is the value of ·397 of a yard? Ans. 1qr. 2·352n.

7. What is the value of ·8469 of a degree? Ans. 58m. 6fur. 35po 0ft. 11in.

8. What is the value of ·569 of a year? Ans. 207d. 16h. 26m. 24ſec.

CASE V.

To find the value of any decimal of a pound by inſpection.

RULE.

Double the firſt figure, or place of tenths, for ſhillings, and if the ſecond figure be 5, or more than 5, reckon another ſhilling; then, after the 5 is deducted, call the figures in the ſecond and third places ſo many farthings, abating 1 when they are above 12, and 2 when above 36, and the reſult will be the anſwer.

NOTE. When the decimal has but two figures, if any thing remain after the ſhillings are taken out, a cypher muſt be annexed to the right hand, or ſuppoſed to be ſo.

EXAMPLES.

1. Find the value of £·876 by inſpection.

16s.=double of 8.
1s. for the 5 in the ſecond place, which is to be taken out of 7,
and 6½d.=26 farthings, remain to be added;
deduct ¼d. for the exceſs of 12.

17s. 6¼d. Ans.

2. Find by inſpection, the value of £·49.

8s.=double of 4.
1s. for the 5 in the place of hundredths.
10d.=40 farthings, a 0 being annexed to the remaining 4.
Ded. ½d. for the exceſs of 36.

9s. 9½d. the anſwer.

2. Find the value of £·097 by inſpection.
Ans. 1s. 11¼d.

4. Value the following decimals, by inſpection, and find their ſum, viz. £·785+£·537+£·916+£·74+£·5+£·25+£·09+£·008. Ans. £3 16s. 6d.

Decimal Tables of Coin, Weight, and Meaſure.

TABLE I. Coin.

£1 the Integer.

Shil.	dec.	Shil.	dec.
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence.	Decimals.
11	·045833
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Farth.	Decimals.
3	·003125
2	·0020833
1	·0010416

TABLE II.

Coin & Long Meas.

1 Shilling & 1 Foot the Integer.

Pence & Inches.	Decimals.
11	·916666
10	·833333
9	·75
8	·666666
7	·583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farthings.	Decimals
3	·0625
2	·041666
1	·020833

TABLE III.

Troy Weight.

1 Pound the Integer. Ounces the ſame as Tab. II.

Penny-weights.	Decimals
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains.	Decimals.
12	·002083
11	·00191
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 Oz. the Integer. Pennyweights the ſame as Shillings in Tab. I.

Grains.	Decimals
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125

Grains.	Decimals.
5	·010416
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.

Avoirdupois Wt.

112lb. the Integer.

Qrs.	Decimals.
3	·75
2	·5
1	·25

Pounds	Decimals.
27	·241071
26	·232143
25	·223214
24	·214286
23	·205357
22	·196428
21	·1875
20	·178571
19	·169643
18	·160714
17	·151786
16	·142857
15	·133928
14	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ounces	Decimals.
15	·008370
14	·007812
13	·007254
12	·006696

Carried over.

Brought over

Ounces.	Decimals.
11	·006138
10	·00558
9	·005022
8	·004464
7	·003906
6	·003348
5	·00279
4	·002232
3	·001674
2	·001116
1	·000558

qrs. of ozs.	Decimals.
3	·000418
2	·000279
1	·000139

TABLE V.
Avoirdupois Wt.
1lb. the Integer.

Ounces.	Decimals.
15	·9375
14	·875
13	·8125
12	·75
11	·6875
10	·625
9	·5625
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

Drams.	Decimals.
15	·058594
14	·054687
13	·050781
12	·046875
11	·042969
10	·039062
9	·035156
8	·031[illegible]
7	·027344
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.
Cloth Measure.
1 Yard the Integer.

Quarters.	Decimals.
3	·75
2	·5
1	·25

Nails.	Decimals.
3	·1875
2	·125
1	·0625

TABLE VII.
Long Measure.
1 Mile the Integer.

Yards.	Decimals.
1000	·568182
900	·511364
800	·454545
700	·397727
600	·340909
500	·284091
400	·227272
300	·170455
200	·113636
100	·056818
90	·051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001705
2	·001136
1	·000568

Feet.	Decimals.
2	·0003787
1	·0001892

Inches.	Decimals
6	·0000947
5	·000079
4	·0000632
3	·0000474
2	·0000316
1	·0000158

TABLE VIII
Liquid Measure
1 Gall. the Integer.
Quarts the ſame as qrs. in Tab. VI.

1 Pint	·125
3 Gill	·09375
2	·0625
1	·03125

TABLE IX.
Time.
1 Year the Integer.
Months the ſame as Pence in Tab. II

Days.	Decimals.
365	1·000000
300	·821918
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164384
50	·136986
40	·109589

Carried over.

Brought over.

Days.	Decimals.
30	·082192
20	·054795
10	·027397
9	·024658
8	·021918
7	·019178
6	·016438
5	·013699
4	·010959
3	·008419
2	·005479
1	·00274

1 Day the Integer.

Hours.	Decimals.
23	·958333
22	·916666
21	·875
20	·833333
19	·791666
18	·75
17	·708333
16	·666666
15	·625
14	·583333
13	·541666
12	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666

Minutes.	Decimals.
30	·020833
20	·013889
10	·006944
9	·00625
8	·005555
7	·004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001389
1	·000694

COMPOUND MULTIPLICATION

IS extremely uſeful in finding the value of goods, &c. And, as in Compound Addition, we carry from the loweſt denomination to the next higher, ſo we begin and carry in Compound Multiplication; one general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c.*— Set down the price of 1, and place the quantity underneath the leaſt denomination, for the multiplier, and in multiplying by it, obſerve the ſame rules for carrying from one denomination to another, as in Compound Addition.

INTRODUCTORY EXAMPLES.

	1.			2.			3.			
	£	s.	d.	£	s	d.	D.	d.	c.	m.
Multiply	15	17	1	25	12	8	8	5	1	7
by			2			3				4

4.

£.	s.	d.
67	18	6½
		5

5.

D.	c.
4	75
	6

6.

£.	s.	d.
13	12	11
		7

7.

£.	s.	d.
31	16	8¼
		8

8.

E.	D.	d.	c.	m.
2	7	8	9	1
				9

Practical Questions.

Note. The facility of reckoning in the Federal Money, compared with pounds, ſhillings, &c. may be ſeen from the examples in this and the following caſes; where the ſame queſtions are given in both the currencies, as near as can be, avoiding ſmall fractions. It may be obſerved, that the variety of caſes here given, is applicable only to the *old* currency, while the ſame queſtions in the Federal are ſolved by plain decimals.

1. What will 9 yards of cloth at $\left\{\begin{array}{l}5s.\ 4d.\\ 88c.\ 9m.\end{array}\right\}$ per yd. come to?

	£0 5 4 price of one yard,	·88c. 9m.
Multiplied by	9 yards	9
Ans.	£2 8 0 price of 9 yards.	D.8 ·00 1

2. 3 yards at $\left\{\begin{array}{l}15s\ 4d.\\ 2D.\ 55c.\ 6m.\end{array}\right\}$ per yd. = $\left\{\begin{array}{l}£2\ \ 6s.\\ D.7\ 66c.\ 8m.\end{array}\right.$

3. 6 - - $\left\{\begin{array}{l}9s.\ 10d.\\ D.1\ 63c.\ 9m.\end{array}\right\}$ - - = $\left\{\begin{array}{l}£2\ \ 19s.\\ D.9\ 83c\ 4m.\end{array}\right.$

4. 9 - - $\left\{\begin{array}{l}13s.\ 7½d.\\ D.2\ 27c.\ 1m.\end{array}\right\}$ - - = $\left\{\begin{array}{l}£6\ \ 2\ \ 7½\\ D.20\ ·43\ 9\end{array}\right.$

CASE II.

When the multiplier, that is, the quantity, is above 12; You muſt multiply by two ſuch numbers, as, when multiplied together, will produce the given quantity.

EXAMPLES.

1. What will 144yds. of cloth coſt at {3s. 5½d.
57c. 6⅔m.}
per yard ?

	£	s.	d.		c. m.
	0	3	5½	price of 1 yard.	·5764
Multiply by			12		144
Produces	2	1	6	price of 12 yards.	23056
Multiplied by			12		23056
					5764
Anſwer	£24	18	0	price of 144 yards.	
				Ans.	D 83·0016

Queſtions.			Anſwers.
2. 24 yards, at	6s. 3¾d. D.1 5c. 2m.	per yd. =	£7 11 6 D 25·24 8
3. 27 ——	9s. 10d. D.1 63c.9m.	——	£13 5 6 D.44·25 3
4. 44 ——	12s. 4½d. D.2 6c. 2½m.	——	£27 4 6 D.90·75

CASE III.

When the quantity is ſuch a number, as that no two numbers in the table will produce it exactly : Then multiply by two ſuch numbers as come the neareſt to it ; and for the number wanting, multiply the given price of 1 yard by the ſaid number of yards wanting, and add the products together for the anſwer ; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which ſubtract from the laſt product, and the remainder will be the anſwer.

EXAMPLES.

1. What will 47 yards of cloth, at {17s. 9d.
D.2 95c. 8m.}
per yard, come to ?

	£0	17	9	price of 1 yard.	D.2·958
Multiplied by			5		47
Produces	4	8	9	price of 5 yards.	20706
Multipliedby			9		11832
Produces	39	18	9	price of 45	Ans. D.139·026
Add	1	15	6	price of 2 yards.	
Ans.	£41	14	3	price of 47 yards.	

Note. This may be performed by firſt finding the value of 48 yards, from which if you ſubtract the price of 1, the remainder will be the anſwer as above.

Queſtions.				Anſwers.
2.	75 yards at	5s. 7½d. 93c. 7½m.	per yd. =	£21 1 10½ D.70·312½
3.	67½ ——	16s. 3¼d. D.2 71c. 1⅘m.	——	£54 18 3¼ D.183·046⅛
4.	59 ——	9s. 7d. D.1 59c. 7m.	——	£28 5 5 D.94·223

CASE IV.

When the quantity is any number above the Multiplication Table: Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then, you muſt multiply the price of 100 by the number of hundreds in in your queſtion; the price of ten by the number of tens; and the price of unity, or 1, by the number of units: Laſtly, add theſe ſeveral products together, and the ſum will be the anſwer.

Examples.

1. What will 359 yards of cloth, at {4s. 7½d. / 77c. 1m.} per yard, amount to?

	£	s.	d.	
	0	4	7½	price of 1 yard.
			10	
	2	6	3	price of 10 yards.
			10	
	23	2	6	price of 100 yards.
			3	
	69	7	6	price of 300 yards.
5 times the price of 10yds.=	11	11	3	price of 50 yards.
9 times the price of 1yd. =	2	1	7½	price of 9 yards.
Ans.	£83	0	4½	price of 359 yards.

c. m.
·771
359

6939
3855
2313

Ans. D.276·789

	Queſtions.			Anſwers.
2.	297 yds. at	17s. 3½d. D.2 88c. 2m.	per yd. =	£256 15 7½ D.855·954
3.	473 —	9s. 11¼d D.1 65c. 6¼m.	—	£235 0 5¼ D 783·406¼
4.	512 —	15s. 10d. D.2 63c. 9m.	—	£405 6 8 D.1351·168

CASE V.

To find the value of one hundred weight: As 112 is the groſs hundred, ſo 112 farthings are =2s. 4d. and 112d.= 9s. 4d.; therefore, if the price be farthings, or not more than 3d. multiply 2s. 4d. by the farthings in the price of 1lb.; or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1lb. and, in either caſe, the product will be the anſwer.

EXAMPLES.

1. What will 1cwt. of chalk come to, at {1½d. / 2c. 1m.} per pound?

1 qr. farthings=£0 2 4 price of 1cwt. at 1qr. per lb.
1½d. = 6 farthings in the price.

Anſwer £0 14 0 price of 1cwt. at 1½d per lb.

·021
112

42
21
21

Anſwer D.2·352

2. 1Cwt. of tin, at $\left\{\begin{array}{c}2\frac{1}{4}\text{d.}\\ 3\text{c. }1\frac{1}{4}\text{m.}\end{array}\right\}$ per pound?

·03125	2s. 4d. price of 1cwt. at ¼d. per lb.
112	9 farthings in the price of 1lb.
6250	£1 1 0 price of 1cwt. at 2¼d. per pound, anſwer.
3125	
3125	
D.3·50000 Ans.	

3. 1Cwt. of lead, at $\left\{\begin{array}{c}7\text{d.}\\ 9\text{c. }8\text{m.}\end{array}\right\}$ per pound?

·098	9s. 4d. price of 1cwt. at 1d. per lb.
112	7 pence in the price of 1lb.
196	£3 5 4 price of 1cwt. at 7d. per lb.
98	
98	
D.10·976 Ans.	

Queſtions.		Anſwers,
4. 1Cwt. at $\left\{\begin{array}{c}0\frac{3}{4}\text{d.}\\ 1\text{c. }0\cdot4\text{m.}\end{array}\right\}$ per pound	=	$\left\{\begin{array}{l}£0\ \ 7\text{s.}\\ \text{D.}1\cdot165\end{array}\right.$
5. —— $\left\{\begin{array}{c}2\frac{1}{2}\text{d. }\frac{1}{8}\text{qr.}\\ 3\text{c. }6\frac{1}{8}\text{m.}\end{array}\right\}$ ——	=	$\left\{\begin{array}{l}£1\ \ 4\ \ 6\\ \text{D.}4\cdot088\end{array}\right.$
6. —— $\left\{\begin{array}{c}4\frac{1}{2}\text{d.}\\ 6\frac{1}{4}\text{c.}\end{array}\right\}$ ——	=	$\left\{\begin{array}{l}£2\ \ 2\text{s.}\\ \text{D.}7\end{array}\right.$

CASE VI.

To find the value of two, or more hundreds, by having the price of one pound: Firſt, find the price of 1cwt. by the laſt caſe, and then proceed to find the value of the whole, by Caſe 1 or 2, as the queſtion may require.

EXAMPLES.

1. What is the value of 5¼cwt. of ſugar, at {6d. / 8c. 3⅓m.} per pound ?

	£	s	d.	
	0	9	4	price of 1cwt. at 1d. per lb.
			6	
	2	16	0	price of ditto at 6d. per lb.
			5	
	14	0	0	price of 5cwt.
1qr. =	0	14	0	price of ¼ cwt.
Ans. =	14	14	0	price of 5¼cwt.

cwt.	qr.	D.
5	1	·083⅓
4		588
21		664
28		664
		415
168		196
42		D.49·000 Ans.
588 lb.		

Queſtions.			Anſwers.
2. 4Cwt. of ſugar, at	{2½d. 3½c.}	per lb. =	{£4 13 4 D.15 68c.
2. 8½ ——	{5d. 7c.}	—— =	{£19 16 8 D.66 64c.
4. 7 ——	{4¾d. 6c. 6m.}	—— =	{£15 10 4 D.51·744

CASE VII.

To find the value of a hundred weight, when the price of 1 lb. *is any number of pounds and ſhillings ; or ſhillings, pence, and farthings :* Multiply the price of 1lb. by 7, its product by 8, and *this* product by 2 ; which laſt product will be the anſwer required.

EXAMPLES.

What will 1cwt. of tobacco coſt, at {5s. 7½d. / 93c. 7½m.} per lb. ?

£	s.	d.		D.
0	5	7½	price of 1lb.	·9375
		7		112
1	19	4½	price of 7lb.	18750
		8		9375
				9375
15	15	0	price of 56lb. or ½ cwt.	
		2		Ans. D.105·
£31	10	0	Ans. price of 112lb. or 1cwt.	

Queſtions.			Anſwers.
2. 1Cwt. at	3s. 10½d. 64c. 6m.	per lb. =	£21 14s. D.72 35c. 2m.
2. 1 ——	9s. 6d. D.1 58⅓c.	— =	£53 4s. D.177 33⅓c.
4. 1 ——	16s. 11½d. D.2 82c. 6m.	— =	£94 19 4 D.316·512

PRACTICAL QUESTIONS IN WEIGHTS AND MEASURES.

1. What is the weight of 4 hogſheads of ſugar; each weighing 7cwt. 3qrs. 19lb. ? Ans. 31cwt. 2qrs. 20lb.

2. What is the weight of 6 cheſts of tea, each weighing 3cwt. 2qrs. 9lb. ? Ans. 21cwt. 1qr 26lb.

3. If I am poſſeſſed of 1½ dozen of ſilver ſpoons, each weighing 3oz. 5pwt.—2 dozen of tea ſpoons, each weighing 15pwt 14gr.—3 ſilver cans, 9oz. 7pwt.—2 ſilver tankards, each 21oz. 15pwt.—and 6 ſilver porringers, each 11oz. 18pwt. ; what is the weight of the whole ? Ans. 18lb. 4oz. 3pwt.

4. In 35 pieces of cloth, each meaſuring 27¾ yards, how many yards ? Ans. 971¼ yards.

5. How much brandy in 9 caſks, each containing 45 gallons, 3qt. 1pt. ? Ans. 412gal. 3qt. 1pt.

6. If I have 9 fields, each of which contains 12 acres, 2 roods, and 25 poles ; how many acres are there in the whole ? Ans. 113ac. 3r. 25p.

COMPOUND DIVISION

IS the dividing of numbers of different denominations: in doing which, always begin at the higheſt, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and ſo on, till you have divided the whole, taking care to ſet down your quotient figures under their reſpective denominations.

INTROUUCTORY EXAMPLES.

1.

	£	s.	d.	
Divide	549	17	9	by 5
Quotient	£109	19	6½	

2.

	D.	d.	c.	m.
3)	14	1	9	6

3.

	£	s.	d.
4)	731	5	10½

4.

	£	s.	d.
6)	37	11	4¼

5.

	D.	c.	m.
7)	25	49	4

6.

	T.	cwt.	qrs.	lb.	oz.	dr.
3)	29	13	2	25	1	13

7.

	lb.	oz.	pwt.	gr.
10)	849	11	12	14

PRACTICAL QUESTIONS.

The ſame note, which was given under Practical Queſtions in Compound Multiplication, is applicable in this place.

CASE I.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 yard: If the quantity do not exceed 12, proceed by ſetting down the price, and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by two ſuch numbers, as, when multiplied together, will produce the quantity, and the laſt quotient will be the value of 1 yard, required.

Note. This Caſe proves the firſt and ſecond Caſes in Compound Multiplication.

Examples.

1. If 9 yards of cloth coſt {£4 3s. 7½d. / D.13 93¾c.} what is it per yard?

£.	s.	d.		D.
9)4	3	7½		9)13'9375
0	9	3½ Ans.		D.1'5486+ Ans.

2. If 7 ells coſt {£5 17s. 5d. / D.19 57c.} what coſt 1 ell?

Ans. {16s. 9¼d. / D.2'796

3. If 11 ſheep coſt {£6 5s. 9d. / D.20 95c. 8m.} what did each coſt?

Ans. {11s. 5¼d. / D.1'905+.

4. If 12 gallons of rum coſt {£8 11s. 9½d. / D.28 63c. 2m.} what is it per gallon?

Ans. {14s. 3¾d. / D.2'386

4. If 84 cows coſt {£253 13s. / D.845 50c.} what is the price of each?

Ans. {£3 0s. 4¾d. / D.10 6½c.

5. If 132 buſhels of corn coſt {£20 12s. 6d. / D.68 75c.} what is that per buſhel?

Ans. {3s. 1½d. / 52c. 1m.

CASE II.

Having the price of a hundred weight, to find the price of 1lb: Divide the given price by 8, that quotient by 7, and this quotient by 2, and the laſt quotient will be the price of 1lb. required.

EXAMPLES.

1. If 1cwt. of flax coſt $\left\{\begin{array}{l}\text{£2 7s. 8d.}\\ \text{D.7 94c. 4m.}\end{array}\right\}$ what is that per pound?

£ s. d.
8)2 7 8
7)0 5 11½
2)0 0 10d. 0$\frac{6}{7}$qr.
0 0 5$\frac{3}{28}$d. price of 1lb.

D.
112)7·944(·071—
784
104
Ans. 7c. 1m. nearly.

2. At $\left\{\begin{array}{l}\text{£3 10s.}\\ \text{D.11 67c.}\end{array}\right\}$ per cwt. what coſt 1lb.?

Ans. $\left\{\begin{array}{l}\text{7½d.}\\ \text{10c. 4m.}\end{array}\right.$

3. At $\left\{\begin{array}{l}\text{£6 6s.}\\ \text{D.21}\end{array}\right\}$ per cwt. what coſt 1lb.?

Ans. $\left\{\begin{array}{l}\text{1s. 1½d.}\\ \text{18c. 7½m.}\end{array}\right.$

4. At $\left\{\begin{array}{l}\text{£42 11s. 8d.}\\ \text{D.141 94c. 4m.}\end{array}\right\}$ per cwt. what coſt 1lb.?

Ans. $\left\{\begin{array}{l}\text{7s. 7¼d.}\\ \text{D.1·267.}\end{array}\right.$

CASE III.

Having the price of ſeveral hundred weight, to find the price per pound: Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the laſt Caſe.

EXAMPLES.

1. If 5cwt. of ſugar coſt $\left\{\begin{array}{l}\text{£13 8s. 4d.}\\ \text{D.44·72c. 2m.}\end{array}\right\}$ what is that per lb.?

£ s. d.
5)13 6 4
8) 2 13 8 price of 1cwt.
7) 0 6 8½ price of 14lb. or ⅛ cwt.
2) 0 0 11½ price of 2lb. or $\frac{1}{56}$ cwt.
0 0 5¼ price of 1lb.

112lb. in a cwt.
5 cwt.

c.m.
56|0)44·722(·079⅞—, or 8 cents nearly, Ans.
392

552
504

$\frac{482}{560} = \frac{7}{8}$ nearly.

2. If 8cwt. of cocoa coſt {£15 17s. 4d. / D.52 88c. 9m} what is that per lb.?
Ans. {4¼d. / 5c. 9m.+

3. If 3¼cwt. of ſugar coſt {£9 17s. 2d. / D.32 86c.} what is it per lb.?
Ans. {6⅛d. / 9c.+

4. If 1¾cwt. of cotton wool coſt {£6 10s. 8d. / D.21 77c. 8m.} what is that per lb.?
Ans. {8d. / 11c. 1m.+

Note. This Caſe proves the 6th in Compound Multiplication.

CASE IV.

Having the price of any number of yards, &c. to find the price of 1 *yard*: Divide the price by the quantity, beginning at the higheſt denomination, and, if any thing remain, reduce it into the next, and every inferiour denomination, and, at each reduction, divide as before, remembering each time to add the odd ſhillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be ¼, ½, or ¾ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed; or, in Federal Money, work by decimals.

EXAMPLES.

1. If 95½lb. of figs coſt $\left\{\begin{array}{l}16l.\ 13s.\ 6\tfrac{3}{4}d.\\ D.55\ 59c.\ 3\tfrac{1}{4}m.\end{array}\right\}$ what are they per lb.?

Quantity=95½ lb.	Price=£16 13 6¾
Mul. by 4	4
Produces 382 for a diviſor.	Product £66 14 3 for a (dividend.

382)66 14 3 (0 3 5¾ $\frac{250}{382}$ per lb.

20	D. c.m.dec. c. m.
382)1334(3	95½=95·5)55·59375(·58 2+ Ans.
1146	4775
188	7843
12	7640
382)2259(5	2037
1910	1910
349	1275
4	
382)1396(3	
1146	
250	

2. If 147 buſhels of rye coſt $\left\{\begin{array}{l}47l.\ 12s.\ 6d.\\ D.158\ 76c.\end{array}\right\}$ what is it per buſhel?

Ans. $\left\{\begin{array}{l}6s.\ 5\tfrac{3}{4}d.\\ D.1\ 8c.\end{array}\right.$

3. If 33¼ yards of baize coſt $\left\{\begin{array}{l}25l.\ 13s.\ 9\tfrac{1}{2}d.\\ D.85\ 63c.\ 2m.\end{array}\right\}$ what is it per yard?

Ans. $\left\{\begin{array}{l}15s.\ 5\tfrac{1}{4}d.\ \tfrac{95}{133}\\ D.2\ 57c.\ 5m.\end{array}\right.$

Note. This proves the 3d and 4th Caſes in Compound Multiplication.

PRACTICAL QUESTIONS IN MONEY.

1. Divide {273l. 9s. 4d. / D.911 55c. 5m.} among 5 men and 4 women, and give the men twice as much as the women.

```
     Men. Wom.
       5 and 4
Mult. by 2
      10 ſhares.
   Add 4 women's ſhares.
      14 the number of equal ſhares in the whole=Diviſor.

               £.   s.  d.    £.   s.  d.
Divide by 14)273   9   4(   19   10   8=1 woman's ſhare.
             14                       4 women.
            ---             -----------
            133              78    2   8=women's ſhare.
            126             -----------
            ---              19   10   8
              7                        2
             20             -----------
            ---              39    1   4=1 man's ſhare.
         14)149(10                     5 men.
            14              -----------
            ---             195    6   8=men's ſhare.
              9              78    2   8=women's ſhare.
             12             -----------
            ---             273    9   4 Proof.
         14)112(8           -----------
            112
D.
14)911·555(65·111+ =1 woman's ſhare.
   84              4 women.
   --      ---------
   71      260·444=women's ſhare.
   70      ---------
   --
    15         65·111+
    14               2
    --        ---------
     15       130·222+ =1 man's ſhare.
     14              5
     --       ---------
      15      651·111+ = men's ſhare.
      14      260·444+ = women's ſhare.
      --
       1      911·555+ Proof.
```

2. Divide { 120l. 17s. 4d. / D 402 88c. [illegible]m. } among 7 men and 7 women, and give the women 3 times ſo much as the men.

Ans. { 4l. 6s. 4d. / D.14 38c. 9m. } =a man's ſhare.
{ 12l. 19s. / D.43 16c. 7m. } =a woman's ſhare.

3. Divide { 39l. 12s. 5d. / D.132 7c. } among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

Ans. { 1l. 1s. 5d. / D.3 57c.— } = a boy's ſhare.
{ 2l. 2s. 10d. / D.7 14c.— } = a woman's ſhare.
{ 4l. 5s. 8d. / D.14 28c.— } = a man's ſhare.

DUODECIMALS:

OR CROSS MULTIPLICATION,

IS a Rule, made uſe of by workmen and artificers in caſting up the contents of their works.

Dimenſions are generally taken in feet, inches, and parts.

Inches and parts are ſometimes called primes, ſeconds, thirds, &c. and are marked thus: inches or primes, ('), ſeconds ("), thirds ('''), fourths (''''); &c.

This method of multiplying is not confined to *twelves*, but may be greatly extended: For any number, whether its inferiour denominations decreaſe from the integer in the ſame ratio, or not, may be multiplied croſs-wiſe; and for the better underſtanding of it, the learner muſt obſerve, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferiour denomination, the value of an unit in the product will be ſo much inferiour

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to the value of an unit in the multiplicand as an unit of the multiplier is leſs than an integer.

Thus, pounds, multiplied by pounds, are pounds ; pounds, multiplied by ſhillings, are ſhillings, &c. ; ſhillings, multiplied by ſhillings, are twentieths of a ſhilling ; shillings, multiplied by pence, are twentieths of a penny ; pence, multiplied by pence, are 240ths of a penny, &c.

RULE.

1. Under the multiplicand write the correſponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the loweſt, by the higheſt denomination in the multiplier, and write the reſult of each under its reſpective term, obſerving, in duodecimals, to carry an unit for every 12, from each lower denomination to its next ſuperiour, and for other numbers accordingly.

3. In the ſame manner multiply all the multiplicand by the primes, or ſecond denomination in the multiplier, and ſet the reſult of each term one place removed to the right hand of thoſe in the multiplicand.

4 Do the ſame with the ſeconds in the multiplier, ſetting the reſult of each term two places to the right hand hand of thoſe in the multiplicand.

5. Proceed in like manner with all the reſt of the denominations, and their ſum will be the anſwer required.

EXAMPLES.

1. Multiply 2½ feet by 2½ feet.

F.	′			Or thus.
2	6			2½
2	6			2½
5	0			5
1	3	0		1¼
Ans. 6	3			Ans. 6¼ ſquare feet = 6 feet

36 inches. So that the 3 is not 3 inches, but 36in. or ¼ of a ſquare foot.

Or thus.
2·5
2·5
125
50
Ans. 6·25 ſquare feet.

2. Multiply 9f. 8′ 6″ by 7f. 9′ 3″.

F.	′	″			
9	8	6			
7	9	3			
67	11	6			=Prod. by the feet in the multiplier.
7	3	4	6‴		=do. by the primes.
	2	5	1	6⁗	=do. by the ſeconds.
75	5	3	7	6	Anſwer.

3. How many ſquare feet in a board 17 feet 6 inches long, and 1 foot 5 inches wide? Ans. 24ft. 10′ 11″

4. How many cubick feet in a ſtick of timber, 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick? Ans. 35ft. 6′ 8″ 6‴.

5. Multiply 3l. 6s. 8d. by 2l. 5s. 7d.

		£	s.	d.
		3	6	8
		2	5	7
£3×£2=£6	=	6	0	0
6s.×£2=12s.	=	0	12	0
8d.×£2=16d.	=	0	1	4
£3×5s.=15s.	=	0	15	0
6s ×5s.=$\frac{30}{20}$s.	=	0	1	6
8d ×5s.=$\frac{40}{20}$d.	=	0	0	2
£3×7d =21d.	=	0	1	9
6s.×7d.=$\frac{42}{20}$d.	=	0	0	2$\frac{1}{10}$
8d.×7d.=$\frac{56}{240}$d.	=	0	0	0$\frac{7}{30}$
	Ans.	7	11	11$\frac{1}{3}$

SINGLE RULE OF THREE DIRECT.

THE Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that ſhall have the ſame proportion to the third, as the ſecond hath to the firſt.

If *more* require *more*, or *leſs*, require *leſs*, the queſtion belongs to the Rule of Three Direct.

But if *more* require *leſs*, or *leſs* require *more*, it belongs to the Rule of Three Inverſe.*

RULE.†

1. State the queſtion by making that number, which aſks the queſtion, the third term, or putting it in the third place: That, which is of the ſame name or quality as the demand, the firſt term; and that which is of the ſame name or quality with the anſwer required, the ſecond term.

---

* *More* requiring *more*, is when the third term is greater than the firſt and requires the fourth term to be greater than the ſecond. And *leſs* requiring *leſs*, is, when the third term is leſs than the firſt, requires the fourth term to be leſs than the ſecond.

Alſo, *more* requiring *leſs*, is when the third term is greater than the firſt, and requires the fourth term to be leſs than the ſecond And *leſs* requiring *more* is when the third term is leſs than the firſt, and requires the fourth term to be greater than the ſecond.

† This Rule, on account of its great and extenſive uſefulneſs, is ſometimes called the *Golden Rule of Proportion*: for, on a proper application of it and the preceding rules, the whole buſineſs of Arithmetick, as well as every mathematical inquiry, depends The rule itſelf is founded on this obvious principle, that the magnitude or quantity of any effect varies conſtantly in proportion to the varying part of the cauſe: Thus, the quantity of goods, bought, is in proportion to the money laid out;—the ſpace, gone over by an uniform motion, is in proportion to the time, &c.

It has been ſhown in Multiplication of Money, that the price of one, multipled by the quantity, is the price of the whole; and in Diviſion, that the price of the whole divided by the quantity is the price of one; Now, in all caſes of valuing goods, &c. where one is the firſt term of the proportion, it is plain that the anſwer, found by this rule, will be the ſame as that found by multiplication of money, and where one is the laſt term of the proportion, it will be the ſame as that, found by Diviſion of Money.

In like manner, if the firſt term be any number whatever, it is plain that the product of the ſecond and third terms will be greater than the true anſwer required, by as much as the price in the ſecond term exceeds the price of one, or as the firſt term exceeds an unit; conſequently this product, divided by the firſt term, will give the true anſwer required.

Note 1 When it can be done, multiply and divide as in Compound Multiplication, and Compound Diviſion.

2. Multiply the ſecond and third numbers together; divide the product by the firſt, and the quotient will be the anſwer to the queſtion, which (as alſo the remainder) will be in the ſame denomination you left the ſecond term

2. If the firſt term, and either the ſecond or third can be divided by any number without a remainder, let them be divided, and the quotient uſed inſtead of them.

The following methods of operation, when they can be uſed, perform the work in a much ſhorter manner than the general rule.

1. Divide the ſecond term by the firſt; multiply the quotient into the third, and the product will be the anſwer.

2. Divide the third term by the firſt; multiply the quotient into the ſecond, and the product will be the anſwer.

3. Divide the firſt term by the ſecond, and the third by that quotient and the laſt quotient will be the anſwer.

4. Divide the firſt term by the third, and the ſecond by that quotient, and the laſt quotient will be the anſwer

Note. The term which aſks or moves the queſtion, has generally ſome words like theſe before it, viz What will? What coſt? How many? How far? How long? How much? &c.

OBSERVATONS.

1. The fourth number is always found in the name in which the ſecond is given, or reduced to; which, if it be not the higheſt denomination of its kind, reduce to the higheſt, when it can be done.

2. When the ſecond number is of divers denominations, bring it to the loweſt mentioned, and the fourth will be found in the ſame name to which the ſecond is reduced, which reduce back to the higheſt poſſible

3. If the firſt and third be of different names, or one or both of divers denominations, reduce them both to the loweſt denomination mentioned in either.

4. When the product of the ſecond and third is divided by the firſt; if there be a remainder after the diviſion, and the quotient be not the leaſt denomination of its kind; then multiply the remainder by that number, which one of the ſame denomination with the quotient contains of the next leſs, and divide this product again by the firſt number; and thus proceed, till the leaſt denomination be found, or till nothing remain

5. If the firſt number be greater than the product of the ſecond and third; then bring the ſecond to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the loweſt name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it moſt generally happens) put the ſeparate content of each properly under one another, then add them together, and you will have the whole quantity.

in, and which may be brought into any other denomination required.

Two or more ſtatings, are ſometimes neceſſary, which may always be known from the nature of the queſtion.

The method of Proof is by inverting the queſtion.

EXAMPLES.

1. If 6lb. of ſugar coſt $\left\{\begin{matrix}9s.\\ D.1\ 50c.\end{matrix}\right\}$ what will 30lb. coſt at the ſame rate?

lb. s. lb.	lb. D. c. lb.
As 6 : 9 :: 30 : the Ans.	As 6 : 1 50 :: 30 : the Ans.
9	30
6)270	6)45·00
45s.=£2 5s. Ans.	D.7 50 Ans.

Here the firſt clauſe (if 6lb. of ſugar coſt 9s. or D.1 50c.) ſuppoſes the rate; then follows the queſtion: What will 30lb. coſt? 30lb. which moves the queſtion, is the 3d term. 6lb. the ſame kind, is the 1ſt, and 9s. (or D.1 50c.) the 2d.

Again. By inverting the order of the queſtion, it will be,

2. If $\left\{\begin{matrix}9s.\\ D\ 1\ 50c.\end{matrix}\right\}$ buy 6lb. of ſugar, how much will $\left\{\begin{matrix}£2\ 5s.\\ D.7\ 50c.\end{matrix}\right\}$ buy at that rate?

s. lb. s.
As 9 : 6 :: 45 : the anſwer.
[illegible]

9)[illegible]70

30lb. Ans.

D. c. lb. D. c.
As 1 50 : 6 :: 7 50 : the anſwer.
6

1·5)45·00(30lb. anſwer.
45

0

Again, 3. If 30lb. of ſugar be worth $\left\{\begin{matrix}\text{£2 5s.}\\ \text{D.7 50c.}\end{matrix}\right\}$ how much may I buy for $\left\{\begin{matrix}\text{9s.}\\ \text{D.1 50c.}\end{matrix}\right\}$?

s. lb. s.
As 45 : 30 :: 9 : the answer.
9

45)270(6lb. the anſwer.
270

D. lb. D.
As 7·50 : 30 :: 1·50 : the anſwer.
30

7·5)45·00(6lb. anſwer.
45 0

Again, 4. Suppoſe $\left\{\begin{matrix}\text{£2 5s.}\\ \text{D.7 50c.}\end{matrix}\right\}$ will buy 30lb. of ſugar, what will 6lb of the ſame ſugar coſt?

lb. s. lb.
As 30 : 45 :: 6 : the anſwer.
6

3|0)27|0

9s. anſwer.

lb. D. lb.
As 30 : 7·50 :: 6 : the anſwer.
6

3|0)4[illegible]

D 1·50 anſwer.

N. B. The three laſt queſtions are only the firſt *varied*, being put merely to ſhow how any queſtion, in this Rule, may be inverted.

5. If 5yds. cloth coſt $\left\{\begin{matrix}\text{£1 10s.}\\ \text{D.5}\end{matrix}\right\}$ what will 20 yards ditto come to?

yds. s. yds.
As 5 : 30 :: 20
30 ÷ 5 = 6

2|0)12|0s.=£6 the anſwer.

Here I divide the 2d term by the 1ſt, and multiply the quotient into the 3d for the anſwer.

yds. s. yds.
Again, 6. As 5 : 30 :: 20
20÷5 = 4

120s.=£6 anſwer.

Here I divide the 3d term by the 1ſt, and multiply the quotient into the 2d, for the anſwer.

Theſe operations would be, perhaps, ſtill more apparent, if performed in Federal Money. Thus:

yds. D. yds.	yds. D. yds.
As 5 : 5 :: 20	As 5 : 5 :: 20
5 ÷ 5 = 1	20÷5 = 4
D.20 anſwer.	D.20 anſwer.

7. If 20 yards coſt D.120, how many yards may I have for D.30?

D. yds. D.
As 120 : 20 :: 30
120÷20=6 quotient, and 30÷6=5 yards, anſwer.

Here I divide the 1ſt term by the 2d, and then, the 3d term by the quotient for the anſwer.

D. yds. D.
Again, 8. As 120 : 20 :: 30
120÷30=[illegible]otient, and 20÷4=5 yards, anſwer.

Here I divide t[illegible]t term by the 3d, and then, the 2d term by that quotient for the anſwer.

9. If 1cwt. of tobacco coſt £5 12 9½, what will 8cwt. ditto coſt?

cwt. £ s. d. cwt.
As 1 : 5 12 9½ :: 8
8

Ans. £45 2 4

Here there is no need of reducing the middle term, becauſe it can be performed by compound multiplication, the firſt term being an unit.

10. If 8cwt. of tobacco coſt £45 2 4, what is that per cwt.?

	£	s.	d.
8)	45	2	4
Ans.	5	12	9½

Here there is no need of reducing the middle term, becauſe it may be performed by compound diviſion only, the 3d term being an unit.

1. If 9cwt. 3qrs. ſugar coſt {£27 17s. 6d. / D.92 91c. 7m.} what will 2cwt. 1qr. 11lb. coſt?

Cwt.	qrs.	£	s.	d.	Cwt.	qr.	lb.
9	3	27	17	6	2	1	11
4		20			4		
39		557			9		
28		12			28		
312		6690			73		
78					19		
1092					263		

lb. d. lb.
As 1092 : 6690 :: 263 : the anſwer.

263
2007
4014
1338
12
1092)1759470(1611[illegible]
1092
2|0)13|4 3d.
6674 £6 14s. 3d. anſwer.
6552
1227
1092
1350
1092
258
4
1092)1032(0qr.

Note 1. If you look at the ſtating, you will ſee that the firſt and third terms are of the ſame kind, but of different denominations, and therefore are reduced to the ſame name or denomination, and that the demand of the queſtion lies on the 3d term.

2. That the middle term, being given in pounds, ſhillings and pence, is reduced to pence. But

3. If the ſecond term were in federal money it would be ſufficient to proceed according to decimals. Thus:

lb. D. lb.
As 1092 : 92·917 :: 263 : the anſwer.
263
______
278751
557502
185834
______ D.c m.
1092)24437·171(22·378+, anſwer.
2184
____
2597
2184
____
4131
3276
____
8557
7644
____
9131
8736
____
395

12. If 57 yards [illegible] $\left\{\begin{matrix}£69\\ D.230\end{matrix}\right\}$ what will 9 yards coſt at that rate?

yds. £ yds.
As 57 : 69 :: 9
9
___
57)621(10l. anſwer.
57
__
51
20
___
1020 Carried over.

Brought over. 57)1020(17s.

57

450

399

51

12

57)612(10d.

57

42

4

57)168($2\frac{54}{57}$qrs.

114

Here, all the terms being whole numbers, there is no need of reducing the middle one until after ſtating.

The ſame in Federal money would ſtand thus:

yds. D. yds.

As 57 : 230 :: 9 : the anſwer.

9

57)2070(36D. 31c. $5\frac{15}{19}$m. Anſwer.

171

360

342

180

171

90

57

330

285

$\frac{45}{57} = \frac{15}{19}$

13. If my income be 109 guineas per annum, I deſire to know what I may ſpend per day, ſo that I may lay up 45l. at the year's end? Ans. 5s. 10$\frac{3}{4}$d. $\frac{1}{365}$ per day.

Note 1. You muſt ſubtract 45l. from the value of 109 guineas.

2. There being 365 days in a year, your queſtion muſt next be ſtated thus:

D. Guin. £ D. s. d. qr.
As 365 : 109—45 :: 1 : 5 10 3$\frac{1}{365}$ the Ans.

14. If my ſalary be 43l. 12s. 5d. per annum, what does it amount to per week? Ans. 16s. 9$\frac{17}{52}$d.

The Stating. W. £ s. d. W.
As 52 : 43 12 5 :: 1 : the anſwer.

Note. As there are 52 weeks and 1 day in a year, you will get the true anſwer to the above queſtion by the following ratio.

D. £ s. d. D.
As 365 : 43 12 5 :: 7 : 16s. 8$\frac{283}{365}$d.

15. Suppoſe my income to be 16s. 8$\frac{283}{365}$d. per week, what is it per annum? Ans. 43l. 13s. 7$\frac{1}{4}$d. $\frac{51}{365}$.

The Stating.

D. s. d. D.
As 7 : 16 8$\frac{283}{365}$ :: 365 : 43l. 12s. 5d. Ans.

16. If I am to pay 1s. 7d. per week for paſturing a cow; what muſt I give per week for 37 cows?

C. s. d. C.
As 1 : 1 7 :: 37 : 2l. 18s. 7d. Ans.

17. How many yards of cloth may be bought for D. 195 75c. of which 9$\frac{1}{2}$yds. coſt 11dol. 2c.?

D. c. yds. D. c. yds. qrs.
A[illegible] 11 02 : 9$\frac{1}{2}$ :: 195 75 : 168 3 Ans.

18. If I buy 57 yards of cloth for 49 guineas; what did it coſt per ell Engliſh?

yds. guin. yds.
As 57 : 49 :: 1$\frac{1}{4}$ 1l. 10s. 1$\frac{13}{228}$d. Ans.

19. A merchant, failing in trade, owes in all £3475, and has in money and effects but £2316 13 4: Now ſuppoſing his effects are delivered up, pray what will each creditor receive on the pound?

£ £ s. d. £
As 3475 : 2316 13 4 :: 1 : £0 13s. 4d. Ans.

20. A owes B 3475l. but B compounds with him for 13s. 4d. on the pound: Pray what muſt he receive for his debt?

£ s. d. £
As 1 : 13 4 :: 3475 : 2316l. 13s. 4d. Ans.

21. If the diſtance from Newburyport to York be 31 miles; I demand how many times a wheel, whoſe circumference is 15$\frac{1}{2}$ feet will turn round in performing the journey?

Feet. Cir. M. Cir.
As 15$\frac{1}{2}$: 1 :: 31 : 10560 times, Anſwer.

22. Bought 9 cheſts of tea, each weighing 3cwt. 2qrs. 21lb. at 4l. 9s. per cwt.; what came they to?

Cwt. £ s. C. qr. lb.
As 1 : 4 9 :: 3 2 21×9 : 147l. 13s. 8$\frac{1}{4}$d.

23. What will 37$\frac{1}{2}$ groſs of buttons come to at 13 cents per dozen?

Doz. c. Groſs. D. c.
As 1 : 13 :: 37$\frac{1}{2}$: 58 50 Ans.

24. A farm, containing 125ac. 3r. 27p. is rented at D.11 500. per acre; what is the yearly rent of that farm?

ac. D. c. ac. r. p. D. c. m.
As 1 : 11 50 :: 125 3 27 : 1447 6 5$\frac{5}{8}$ Ans.

25. If a ſhip coſt 537l. what are $\frac{3}{8}$ of her worth?

Eight. £ Eight.
As 8 : 537 :: 3 : 201l. 7s. 6d. Ans.

26. If $\frac{7}{16}$ of a ſhip coſt 1163D. what is the whole worth?

Sixt. D. Sixt. D. c. m.
As 7 : 1163 :: 16 : 2[illegible]58 28 5 Ans.

27. Bought a caſk of wine at 76c. 5m. per gallon, for 125 dollars; how much did it contain;

Ans. 163gal. 1qt. 1$\frac{3}{17}$pt.

28. What comes the inſurance of 537l. 15s. to at 4$\frac{1}{2}$l. per centum?

£ £ £ s. £ s. d.
As 100 : 4$\frac{1}{2}$ :: 537 15 : 24 2 11$\frac{1}{2}$ $\frac{8}{10}$ Ans.

29. What come the commiſſions of 785l. to at 3$\frac{1}{2}$ guineas per cent.? Ans. 38l. 9s. 3$\frac{1}{2}$ $\frac{4}{10}$d.

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards: each package contained 8 parcels, each parcel, 12 pieces, and each piece, 20 yards: how many dollars came the whole to, and how many per yard?

Yds. guin. pack. D.
As 7 : 3 :: 9 : 34560 Ans. for the whole coſt.
Yds. guin. yd. D.
As 7 : 3 :: 1 : 2 Ans. per yard.

31. A merchant bought 49 tuns of wine for D.910; freight coſt D.90; duties, D.40; cellar, D.31 67c.; other charges D.50, and he would gain D.185 by the bargain; what muſt I give him for 23 tuns?

Tuns. D. D. D. D.c. D. Tuns. D.
As 49 : 910+90+40+37 67+185 :: 23 : 613 33c. Ans.

32. If D.100 gain D.6 in a year, what will D 475 gain in that time? Ans. D 28 50c.

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 ſtatute miles?

H. D. M. M. M.
As 24 : 360×51 :: 1 : $12\frac{3}{4}$ Ans.

34. Shipt for the Weſt Indies 225 quintals of fiſh, at 15s. 6d. per quintal; 37000 feet of boards, at $8\frac{1}{3}$doll. per 1000; 12000 ſhingles, at $\frac{1}{2}$guin. per 1000; 19000 hoops, at $1\frac{1}{2}$doll. per 1000, and 53 half joes; and in return, I have had 3000gals. of rum, at 1s. 3d. per gallon; 2700 gallons of molaſſes at $5\frac{1}{2}$d. per gallon; 1500lb. of coffee, at $8\frac{1}{2}$d. per lb.; and 19Cwt. of ſugar, at 12s. 3d. per cwt. and my charges on the voyage were 57l. 12s.: Pray, did I gain or loſe, and how much, by the voyage?

Ans. loſt £134 9 9.

35. If a ſtaff, 4 feet long, caſt a ſhade (on level ground) 7 feet; what is the height of that ſteeple, whoſe ſhade, at the ſame time, meaſures 198 feet?

F. ſh. F. hei. F. ſh. F. hei.
As 7 : 4 :: 198 : $113\frac{1}{7}$ Ans.

*36. Suppoſe a tax of D.755 be laid on a town, and the inventory of all the eſtates in the town amounts to 9345 dollars, what muſt A pay, whoſe eſtate is D.149?

D. D. D. D. c. m.
As 9345 : 755 :: 149 : 12 12 7 Ans.

* It may not be amiſs to ſhow the general method of aſſeſſing town or pariſh taxes. Firſt, then, an inventory of the value of all the eſtates, both real and perſonal, and the number of polls, for which each perſon is rateable, muſt be taken in ſeparate columns: The moſt conciſe way is then to make the total value of the inventory the firſt term, the tax to be aſſeſſed, the ſecond, and D.1 the third, and the quotient will ſhow the value on the dollar: 2dly, Make a table, by multiplying the value on the dollar by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and perſonal eſtates of each man, and find them ſeparately in the table, which will ſhew you each man's proporrional ſhare of the tax for real and perſonal eſtates.

Note. If any part of the tax is averaged on the polls, or otherwiſe, before ſtating to find the value on the dollar, you muſt deduct the ſum of the average tax from the whole ſum to be aſſeſſed: for which average, you muſt have a ſeparate column, as well as for the real and perſonal eſtates.

EXAMPLE.

Suppoſe the General Court ſhould grant a tax of D.500000, of which the town of Newburyport is to pay D.5312 50c. and, of which the polls, being 1550, are to pay D 1 25c. each:—The town's inventory amounts to D.450000; what will it be on the dollar, and what is A's tax, whoſe eſtate, (as by the inventory) is as follows, viz. real D.1376, perſonal D.1149, and he has 3 polls?

Pol. D. c. Pol. D. c.
Firſt, As 1 : 1 25 :: 1550 : 1937 50 the average part of the tax to be deducted from D.5312 50c. and there will remain D.3375.

D. D. D.
Secondly, As 45000 : 3375 :: 1 : 7½m. on the dollar.

TABLE.

D.		D.	c.	m.	D.		D.	c.	m.	D.		D.	c.
1	is	0	0	7½	20	is	0	15	0	200	is	1	50
2	—	0	1	5	30	—	0	22	5	300	—	2	25
3	—	0	2	2½	40	—	0	30	0	400	—	3	00
4	—	0	3	0	50	—	0	37	5	500	—	3	75
5	—	0	3	7½	60	—	0	45	0	600	—	4	50
6	—	0	4	5	70	—	0	52	5	700	—	5	25
7	—	0	5	2½	80	—	0	60	0	800	—	6	00
8	—	0	6	0	90	—	0	67	5	900	—	6	75
9	—	0	6	7½	100	—	0	75	0	1000	—	7	50
10	—	0	7	5									

Now, to find what A's rate will be

37. If 52 gallons of water in one hour, fall into a ciſtern, containing 230 gallons, and by a pipe in the ciſtern 35 gallons run out in an hour; in what time will it be filled?

Gal. gal. h. gal. h.
As 50—35 : 1 :: 230 : 15$\frac{1}{3}$ Ans.

38. A butcher went with £416, to buy cattle: Oxen, at £22 each, cows at £4, ſteers at £3 10s. and calves at £2 10s. and of each a like number; how many of each could he purchaſe with that ſum?

£ £ £ s £ s. each. £ each.
As 22+4+3 10+2 10 : 1 :: 416 : 13 Ans.

39. Said Harry to Dick, my purſe and money are worth 3$\frac{1}{4}$ guineas but the money is worth eleven times as much as the purſe; pray, how much money is there in it?

Guin. s d.
As 12 : 1 :: 3$\frac{1}{4}$: 7 7
Then £4 11s.—7s. 7d.=£4 3s. 5d. Ans.

40. How many dozen pair of gloves, at 13 groats per pair, may I have for 125 dollars?

Gr. pr. dol. doz. pr.
As 13 : 1 :: 125 : 14 5$\frac{4}{31}$ Ans.

41. There is a ciſtern, having four cocks; the firſt will empty it in ten minutes; the ſecond in 20 minutes; the third in 40, and the fourth in 80 minutes; in what time will all four running together, empty it?

His real eſtate being D.1376, I find, by the table,
That D.1000 is D.7 50c.
That D. 300 is 2 25
That 70 is 52 5m.
And that 6 is 4 5

For his real eſtate, D 10 32

In like manner, I find his tax for perſonal eſtate to be } D.8 61 7½
His three polls, at D.1 25c. each are 3.75 } + D 10 32 = D.22 68c. 7½m. or D.22 69c. Ans.

Real.			Perſonal.			Polls.			Total.		
D.	c.	m.	D	c.	m	D.	c.	m	D	c	m.
10	32	0	8	61	7½	3	75		22	68	7½

$$\text{As}\left\{\begin{array}{c}10\\20\\40\\80\end{array}\right\} : \overset{\text{Ciſt.}}{1} :: \overset{\text{Min.}}{60} : \left\{\begin{array}{c}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{array}\right\}$$

$$\overline{11\frac{1}{4}}$$

Ciſt. Min. Ciſt. Min.
As $11\frac{1}{4}$: 60 :: 1 : $5\frac{1}{3}$ Ans.

42. A and B depart from the ſame place, and travel the ſame road: but A goes 5 days before B, at the rate of 20 miles per day; B follows at the rate of 25 miles per day: In what time and diſtance will he overtake A?

M. M. D. M. D. D. D. M. D. M.
As 25—20 : 1 :: 20×5 : 20. And, As 1 : 25 :: 20 : 500

43. Sold a cargo of flaxſeed in Ireland, for £1795 10s. Iriſh money; what does that amount to, in Maſſachuſetts currency, £81 5s. Iriſh being equal to £100 Maſſachuſetts?

Iriſh. Maſſ. Iriſh. Maſſ.
As £$81\frac{1}{4}$: £100 :: £$1795\frac{1}{2}$: £2209 16s. 11d. Ans.
Or, As £13 : £$1795\frac{1}{2}$ :: £16 : £2209 16s. 11d. as before, becauſe £13 Iriſh are equal to £16 Maſſachuſetts.

44. My correſpondent in Maryland purchaſed a cargo of flour for me, for 437l. that currency; how much Maſſachuſetts money muſt I remit him, 125l. Maryland being equal to 100l. Maſſachuſetts, or 5 Mar.=4 Maſſ?

Ans. 349l. 12s Maſſ.

45. A bill of exchange was accepted at Newburyport for the payment of 345l. 10s. for the like value delivered in New-York, at $133\frac{1}{3}$l. New York currency for 100l. Maſſachuſetts ditto; how much money was paid in New-York?

Maſſ. N. Y. Maſſ. N. Y.
As 75l. : 100l. :: 345l. 10s. : 460l. 13s. 4d. Ans.

46. When the exchange from Maſſachuſetts to Georgia is $83\frac{1}{3}$l. Georgia per 100l. Maſſachuſetts, how much Maſſachuſetts money muſt be paid in Boſton to balance 457l. Georgia currency? Ans 548l. 8s. Maſſ.

47. A merchant delivered at Boſton, 320l. Maſſachuſetts currency, to receive 400l in Philadelphia; what was the Maſſachuſetts pound valued at?

Ans. 1l. 5s. Phil.

48. If I draw a bill of exchange for 537l. 10s. 6d. Maſſachuſetts, to be paid in Ireland, at 123$\frac{1}{13}$l. Maſſachuſetts, per 100l. Iriſh; for how much Iriſh money muſt I draw the bill? Ans. 436l. 14s. 9$\frac{1}{4}$d. Iriſh.

49 Suppoſe a bill is drawn in Ireland, and payable in Boſton, for 673l. 12s. 6d. Iriſh; how much Maſſachuſetts money comes it to, the exchange at 81$\frac{1}{4}$l Iriſh, per 100l. Maſſachuſetts? Ans. 829l. 1s. 6$\frac{6}{13}$d. Maſſ.

50. If I give D.1 75c. for 3 yards, how many yards may I have for D.180? Ans 308yds. 2qrs. 1$\frac{1}{7}$n.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.*

Having made the neceſſary preparations, as directed in Multiplication and Diviſion of Vulgar Fractions, ſtate your queſtion as in whole numbers, and invert the firſt term of the proportion; then multiply the three terms continually together, and the product will be the anſwer.

EXAMPLES.

1. If $\frac{5}{8}$ of a yard coſt $\frac{5}{7}$ of a £. what will $\frac{9}{15}$ of an Ell Engliſh coſt?

$$\tfrac{5}{8}\text{yd.}=\tfrac{5}{8}\text{ of }\tfrac{4}{1}\text{ of }\tfrac{1}{5}=\frac{5\times4\times1}{8\times1\times5}=\tfrac{1}{2}\text{ ell Eng.}$$

E. Eng. £ E. Eng.

$$\text{As }\tfrac{1}{2} : \tfrac{5}{7} :: \tfrac{9}{13} : \tfrac{2}{1}\times\tfrac{5}{7}\times\tfrac{9}{13}=\frac{2\times5\times9}{1\times7\times15}=\tfrac{90}{105}\text{l.}=17\text{s. }1\tfrac{1}{2}\text{d.}$$

$\frac{6}{7}$ anſwer.

2. If $\frac{3}{5}$yd. coſt $\frac{7}{8}$dol. what will 40$\frac{1}{2}$ yards come to?
Ans. D.59 6c. 2$\frac{1}{2}$m.

3. A perſon having $\frac{4}{5}$ of a veſſel, ſells $\frac{2}{3}$ of his ſhare for D.1080$\frac{2}{3}$; what is the whole veſſel worth?
Ans. D.2026 25c.

4. A merchant ſold 5$\frac{1}{2}$ pieces of cloth, each containing 12$\frac{2}{3}$yds. at 12$\frac{2}{3}$c. per yard; what did the whole amount to?
Ans. D.8 82$\frac{4}{9}$c.

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* This rule and the next, depend upon the ſame principle as the Rule of Three in whole numbers.

5. A buys of B 560$\frac{3}{4}$l. bank ſtock, at 85$\frac{2}{3}$l. per cent. ; what comes it to ? Ans. 480l. 7s. 6$\frac{1}{5}$d.

6. A merchant bought a number of bales of velvet, each containing 129$\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and ſold them out at the rate of 11 dollars for 7 yards ; and gained 200 dollars by the bargain ; how many bales were there ?

yds. Dol. yds. Dol.

As 7 : 11 :: 5 : 6$\frac{6}{7}$

Sold 5 yards for 7$\frac{6}{7}$ dollars.

Bought 5 yards for 7 dollars.

In 5 yards gained $\frac{6}{7}$ dollar.

Dol. yds. Dol. yds.

As $\frac{6}{7}$: 5 :: 200 : 1166$\frac{2}{3}$, and,

yds. B. yds. B.

As 129$\frac{17}{27}$: $\frac{1}{1}$ :: 1166$\frac{2}{3}$: 9 anſwer.

Although the method before laid down be univerſally applicable, yet there are other methods more ready and expeditious in ſome particular caſes.

Rule I.

If the firſt and third terms be fractions, and the ſecond a whole number, reduce the firſt and third to one common denominator, then, rejecting the denominators, make the numerator of the firſt, the firſt term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{8}$ yard coſt 9s. what coſt $\frac{7}{12}$ yard at that rate ?

$\frac{5}{8}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, As 15 : 9s. :: 14 : 8s. 4$\frac{3}{4}$d. Ans.

Rule II.

If of the firſt and third terms, one be 1, and the other a fraction ; put the denominator of the fraction inſtead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land coſt 12l. what will $\frac{5}{8}$ of an acre coſt at that rate ?

Den. £ Num. £. s.

As 8 : 12 :: 5 : 7 10 Ans.

RULE III.

If the ſecond term be a fraction likewiſe, (that is, if all the terms be fractions) having reduced the firſt and third to one common denominator, multiply the numerator of the firſt term by the denominator of the ſecond, for a diviſor; and the numerator of the third by the numerator of the ſecond, for a dividend; divide the laſt product by the firſt, and the quotient will be the anſwer.

If $\frac{1}{2}$ yard of cloth coſt £$\frac{3}{4}$ what coſt $\frac{7}{8}$ yard?

$\frac{1}{2}=\frac{4}{8}$, which reduces it to a common denominator; then

As $4 : \frac{3}{4} :: 7$

$$\begin{array}{r}4\\4\\\hline 16\end{array}\qquad\begin{array}{l}7\\3\\\hline 16)21(1\tfrac{5}{16}l.=26s.\ 3d.\ \text{Ans.}\\16\\\hline5\end{array}$$

RULE OF THREE DIRECT IN DECIMALS.

RULE.

Having reduced your fractions to decimals, and ſtated your queſtion as in whole numbers, multiply the ſecond and third together; divide by the firſt, and the quotient will be the anſwer.

EXAMPLES.

1. If $\frac{5}{8}$ of a yard coſt $\frac{7}{12}$ of a pound; what will $9\frac{2}{3}$ yards come to?

$\frac{5}{8}=\cdot625$, $\frac{7}{12}=\cdot583+$, and $\frac{2}{3}=\cdot667-$.

As ·625 : ·583 :: 9·667

$$\begin{array}{r}9\cdot667\\\cdot583\\\hline 29001\\77336\\48335\\\hline\end{array}$$

$$\begin{array}{l}\cdot625)5635861(9\cdot017+=9l.\ 0s\ 4d.\ \text{Ans.}\\5625\\\hline1086\\625\\\hline4611\\4375\\\hline236\end{array}$$

2. If 1oz. of ſilver coſt 6s. 8d. what is the price of a bowl, which weighs 1lb. 7oz. 13gr. Ans. 6l. 6s. 10d.

3. If 9$\frac{3}{4}$ yards coſt D.11 25c. what will $\frac{1}{2}$ yard come to? Ans. 57c. 6$\frac{12}{13}$m.

4. There is a ciſtern, which has 3 cocks, the firſt will emyty it in $\frac{1}{4}$ of an hour, the ſecond in $\frac{3}{4}$, and the third in 1$\frac{1}{2}$ hour: In what time will it be emptied if all three run together?

As ·25 : 1 :: 1 : 4
·75 : 1 :: 1 : 1·333+
1·5 : 1 :: 1 : 0·667—

6 Ciſterns

Ciſt. ho. Ciſt.
As 6 : 1 :: 1 : ·1667—= 10min. Ans.

5. A conduit has a cock, which running into a ciſtern, will fill it in 12 minutes: This ciſtern has three cocks; the firſt will empty it in 1$\frac{1}{4}$ of an hour, the ſecond in 37$\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the ciſtern be filled, if all four run together?

As ·2 : 1 :: 1 : 5 the filling cock.
1·25 : 1 :: 1 : 0·8
·625 : 1 :: 1 : 1·6 emptying cocks.
·5 : 1 :: 1 : 2

4·4

5 ciſterns filled in an hour.
4·4 ditto emptied in do.
·6 ciſtern difference.

Ciſt. h. Ciſt.
Then, as ·6 : 1 :: 1 : 1·67— = 1h. 40m. Ans.

6. If 19 yards coſt D.25·75, what will 435$\frac{1}{2}$ yards come to?

yds. D. d. c. yds.
As 19 : 25· 7 5 :: 435·5
25·75

21775
30485
21775
8710

D. d. c. m.
19)11214·125(590· 2 1 7$\frac{2}{19}$ Ans.

7. If 345 yards of tape coſt D.5· 1d. 7c. 5m. what will 1yd. coſt?

yds. D. yds. c.m.
As 345 : 5·175 :: 1 : ·015 Ans.

8. If I give D.12 8d. 2c. 5m. for 675 tops, how many tops will 19 mills buy? Ans. 1 top.

RULE OF THREE INVERSE,
OR RECIPROCAL PROPORTION,

TEACHES, by having three numbers given to find a fourth, which ſhall have the ſame proportion to the ſecond, as the firſt has to the third.

Therefore, the greater the third term is, in reſpect to the firſt, the leſs will the fourth term be, in reſpect to the ſecond; or, the leſs the third term is in proportion to the firſt, the greater the fourth muſt be in proportion to the ſecond; and this is called *reciprocal*, *inverted*, or *indirect* Proportion.

The principal difficulty that will embarraſs the learner will be, to diſtinguiſh when the proportion is direct, and when indirect. This is done by an attentive conſideration of the ſenſe and tenour of the queſtion propoſed: For if thereby it appears that, when the third term of the ſtating is leſs than the firſt, the anſwer muſt be leſs than the ſecond; or, when the third is greater than the firſt, the anſwer muſt be greater than the ſecond; then the proportion is direct: But, if the third be leſs than the firſt, and yet the ſenſe of the queſtion requires the fourth to be greater than the ſecond; or the third, being greater than the firſt, the anſwer muſt be leſs than the ſecond, the proportion is inverſe.

RULE.*

State and reduce the terms as in the Rule of Three Direct; then, multiply the firſt and ſecond terms together, and divide the product by the third; the quotient will be the anſwer in the ſame denomination as the middle term was reduced into.

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* The reaſon of this rule may be explained from the principles of Compound Multiplication and Compound Diviſion, in the ſame

If there be fractions in your queſtion, they muſt be ſtated as before directed, and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the anſwer.

EXAMPLES.

1. How much ſhalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of cloth which is $1\frac{1}{4}$ yard wide?

$$\begin{array}{lccc} & \text{yd.} & \text{yds.} & \text{qrs.} \\ \text{As} & 1\frac{1}{4} : & 6\frac{3}{4} :: & 3 \\ & 4 & 4 & \\ & \overline{5} & \overline{27} & \end{array} \qquad \begin{array}{lrrr} & \text{qrs.} & \text{qrs:} & \text{qrs.} \\ \text{As} & 5 : & 27 :: & 3 \\ & & 5 & \\ & & 3)\overline{135} & \\ & & 4)\overline{45} & \\ & & \overline{11\frac{1}{4}} & \text{yards, Ans.} \end{array}$$

The ſame by Vulgar Fractions.

Firſt, $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3qrs.$=\frac{3}{4}$. Then,

As $\frac{5}{4} : \frac{27}{4} :: \frac{3}{4}$. And $\frac{5}{4}\times\frac{27}{4}\times\frac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\frac{540}{48}=\frac{45}{4}=11\frac{1}{4}$ yds. the anſwer.

---

manner as the direct rule.—*For example*, If 4 men can do a piece of work in 12 days, in what time will 8 men do it?

As 4 men : 12 days :: 8 men : $\frac{4\times12}{8}$=6 days, the Anſwer.

And here the product of the firſt and ſecond terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one eighth part of the time, or 6 days.

The ſame by Decimal Fractions.

$1\frac{1}{4}=1\cdot25$, $6\frac{3}{4}=6\cdot75$, and 3qrs.$=\cdot75$. Then

```
As 1·25 : 6·75 :: ·75
          1·25
          ----
          3375
         1350
         675
       -------
·75)8·4375(11·25 yards, Anſwer.
    75
    --
     93
     75
     --
     187
     150
     ---
      375
      375
```

2. What length of board, 7 inches wide, will make a ſquare foot?

	In. br.		in. len.		in. br.		in.len.	
As	12	:	12	::	$7\frac{1}{2}$	:	$19\frac{1}{5}$	Ans.

3. How many yards of carpet, $2\frac{1}{4}$ feet wide will cover a floor which is 18 feet long and 16 feet wide?

	ft.		ft.		ft. ft.		yds.	
As	16	:	18	::	$2\frac{3}{4}\times3$	:	$34\frac{10}{11}$	Ans.

NOTE, I multiply $2\frac{3}{4}$ by 3, becauſe 3 feet=1 yard.

4. Suppoſe I lend a friend £350 for 5 months, he promiſing the like kindneſs; but, when requeſted, can ſpare but £125; how long may I keep it to balance the favour?

	£		Mo.		£		Mo.	
As	350	:	5	::	125	:	14	Ans.

5. Suppoſe 450 men are in a garriſon, and their proviſions are calculated to laſt but 5 months; how many muſt leave the garriſon, that the ſame proviſions may be ſufficient for thoſe who remain 9 months?

	Mo.		M.		Mo.		M.	
As	5	:	450	::	9	:	250,	and 450—250=200 men Ans.

6. If a man perform a journey in 15 days, when the day is 12 hours long, in how many days will he do it, when the day is but 10 hours? Ans. 18 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide muſt it be, when it is but 19 rods long, to make an acre? Ans. 8 rods 6ft. 11$\frac{7}{19}$in.

8. If when wheat is D. 1 per buſhel, the two penny loaf weigh 9·6oz. what ought it to weigh, when wheat is D. 1 25c. per buſhel? Ans. 7oz. 13pwt. 14·4gr.

9. If a piece of board be 30 inches in length, what breadth will make 1$\frac{1}{4}$ ſquare foot? Ans. 7·2 inches.

10. If 9 men can build a houſe in 5 months, by working 14 hours per day, in what time will the ſame number of men do it, when they work only 10 hours per day?
Ans. 7 months.

11. A wall, which was to be built 24 feet high, was raiſed 8 feet by 6 men, in 12 days: How many men muſt be employed to finiſh the wall in four days?

ft. m. ft. m.
As 8 : 6 :: $\overline{24-8}$: 12 to finiſh it in 12 days; and
d. m. d. m.
As 12 : 12 :: 4 : 36 to finiſh in 4 days.

12. There is a ciſtern having a pipe, which will empty it in 6 hours: How many pipes of the ſame capacity, will empty it in 20 minutes?

h. pi. mi. pi.
As 6 : 1 :: 20 : 18 Ans.

13. What number of men muſt be employed to finiſh in 9 days what 15 men would be 30 days about?
Ans. 50 men.

14. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

15. How much in length, that is 8$\frac{5}{7}$ inches broad, will make a foot ſquare? Ans. 16$\frac{32}{61}$ inches.

16. How much in length that is 13$\frac{7}{8}$ poles in breadth, will make a ſquare acre? Ans. 11$\frac{59}{111}$ poles.

17. A regiment of ſoldiers, conſiſting of 745 men, is to be clothed, each ſuit to contain 3$\frac{1}{2}$ yards of cloth, which

is $1\frac{3}{8}$ yard wide, and lined with ſhalloon $\frac{7}{8}$ yard wide; how many yards of ſhalloon will line them?

As $\overline{745\times3\frac{1}{2}}$: $1\frac{3}{8}$ :: $\frac{7}{8}$: $4097\frac{1}{2}$ yards, Ans.

18. If a ſuit of clothes can be be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the ſame perſon?

Ans. 6yds. 1qr. $3\frac{5}{7}$n.

COMPOUND PROPORTION,

OR DOUBLE RULE OF THREE,

TEACHES to reſolve ſuch queſtions as require two, or more, ſtatings by ſimple proportion; and that, whether direct or inverſe: It is compoſed (commonly) of 5 numbers to find a ſixth, which if the proportion be direct, muſt bear ſuch proportion to the 4th and 5th as the 3d bears to the 1ſt and 2d; but if inverſe, the 6th number muſt bear ſuch proportion to the 4th and 5th, as the firſt bears to the 2d and 3d.

RULE.

Always place the three conditional terms in this order: That number, which is the principal cauſe of gain, loſs or action, poſſeſſes the firſt place; that, which denotes the ſpace of time, diſtance of place, rate, medium or mean of action, the ſecond; and that, which is the gain, loſs or action, the third: This being done, place the other two terms which move the queſtion, under thoſe of the ſame name, and if the blank place, or term ſought, fall under the third place, then the queſtion is in direct proportion: there[illegible]e,

Rule 1.—Multiply the three laſt terms together, for a dividend, and the two firſt for a diviſor:—But if the blank fall under the firſt or ſecond place; then, the proportion is inverſe; therefore,

Rule 2.—Multiply the firſt, ſecond and laſt terms together for a dividend, and the other two for a diviſor, and the quotient will be the anſwer.

EXAMPLES.

1. If D.100 gain D.6 in a year; what will D.400 gain in 9 months?

D.P. Mo. D. Int.
100 : 12 :: 6 { Terms in the ſuppoſition, or conditional terms.
400 : 9 Terms which move the queſtion.

Here, the blank falling under the third place, the queſtion is in direct proportion, and the anſwer muſt be found by the firſt Rule; therefore,

400× 9×6=21600 For the dividend, and,
100×12 =1200 For the diviſor.

See the work at large.

D. Pr. Mo. D. Int.
100 : 12 :: 6
400 : 9
9

100 3600
12 6

12|00)216|00(18D. Ans.
12

96
96

2. If D.100 will gain D.6 in a year; in what time will D.400 gain D.18?

D. Mo. D.
100 : 12 :: 6 Terms in the ſuppoſition.
400 : :: 18 Terms which move the queſtion.

Here the blank falling under the 2d place, the queſtion is in reciprocal or inverſe Proportion, and the anſwer muſt be ſought by the ſecond rule: therefore,

100×12×18=21600 For the dividend.
400× 6 = 2400 For the diviſor.

D.Pr. Mo. D.Int.
100 : 12 :: 6
400 : :: 18
6 12

2400 216
100

24|00)216|00(9 months, Ans.
216

3. What principal, at 6 per cent. per ann. will gain D.18 in 9 months?

Pr. Mo. Int.
100 : 12 :: 6
9 :: 18
12

9 216
6 100
D.
54)21600(400 Ans.
216

Here the blank falling under the firſt place, the proportion is inverſe, and the anſwer found by the ſecond rule, as in the laſt example.

4. If D.400 gain D.18 in 9 months; what is the rate per cent. per annum?

Pr. Mo. Int.
400 : 9 :: 18
100 : 12 :: D.6 Ans.

5. If 8 men ſpend £32 in 13 weeks; what will 24 men ſpend in 52 weeks? Ans. £384.

6. If the freight of 9hhds. of ſugar, each weighing 12cwt. 20 leagues, coſt D.50; what muſt be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$cwt. 100 leagues? Ans. D.189 35c. $1\frac{23}{27}$m.

$289, 35, 1 [illegible]

7. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick?

m. da. ft.
6 : 16 :: 20×6×4
24 : :: 200×8×6 80 days, Ans.

COMPARISON OF WEIGHTS AND MEASURES.

Examples.

1. If 78 pence Maſſachuſetts be worth one 1 French crown, how many Maſſachuſetts pence are worth 320 French crowns?

F. cr. d. F. cr.
As 1 : 78 :: 320
78

2560
2240

24960 Ans.

1. If 24 yards at Boſton make 16 ells at Paris, how many ells at Paris will make 128 yards at Boſton?

Boſt. Par. Boſt. Par.
As 24yds. : 16ells :: 128yds : 85$\frac{1}{3}$ells, Ans.

2. If 60lb. at Boſton make 56lb. at Amſterdam, how many lb. at Boſton will be equal to 350 at Amſterdam?

Ans. 375lb. Boſton.

4 If 95lb. Flemiſh make 100lb. American, how many American lbs. are equal to 550lb. Flemiſh?

Ans. 578$\frac{18}{19}$lb. American.

CONJOINED PROPORTION

IS when the coins, weights or meaſures of ſeveral countries are compared in the ſame queſtion; or, in other words, it is joining many proportions together, and by the relation, which ſeveral antecedents have to their conſequents, the proportion between the firſt antecedent and the laſt conſequent is diſcovered, as well as the proportion between the others in their ſeveral reſpects.

This rule may generally be ſo abridged by cancelling equal quantities on both ſides, and abbreviating commenſurables, that the whole operation may be performed with very little trouble, and it may be proved by as many ſtatings in the Single Rule of Three, as the nature of the queſtion may require.

CASE I.

When it is required to find how many of the firſt ſort of coin, weight or meaſure, mentioned in the queſtion, are equal to a given quantity of the laſt.

RULE.

Place the numbers alternately, that is, the antecedents at the left hand, and the conſequents at the right, and let the laſt number ſtand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a diviſor, and the quotient will be the anſwer.

EXAMPLES.

1. Suppoſe 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouſe, and 100 canes of Thoulouſe=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many yards of America are equal to 379 ells of Hamburgh?

Antecedents.		Conſequents.	Abridged.	
			Ant.	Con.
100 of America	=	100 of England.		
100 of England	=	50 of Thoulouſe.	5	8
100 of Thoulouſe	=	160 of Geneva.	379	
100 of Geneva	=	200 of Hamburgh.		
379 of Hamburgh?				

Therefore, $\frac{379\times5}{8}=236\frac{7}{8}$yds. of America=379 ells of Hamburgh.

ILLUSTRATION.

The two 100s of both ſides cancel each other. Let the laſt cyphers of the three next antecedents and conſequents be cancelled, which is dividing by 10. Then divide the ſecond antecedent and conſequent by 5, and the quotients will be 2 on the ſide of the antecedents, and 1 on the ſide of the conſequents; then 2 will meaſure the third antecedent and conſequent, and the quotients will be 5 and 8. 10 will meaſure the 4th antecedent and conſequent, and the quotients will be 1 and 2. Now, there being two left on each ſide, they cancel each other, and as there is no farther room for abridging by reaſon of that odd number 379, the operation is finiſhed, and the anſwer found as before.

2. If 12lb. at Boſton make 10lb. at Amſterdam, and 100lb. at Amſterdam 120lb. at Paris: How many lb. at Boſton are equal to 80lb. at Paris? Ans. 80lb.

3. If 40lb. at Newburyport make 36 at Amſterdam, and 90lb. at Amſterdam make 116 at Dantzick: How many lb. at Newburyport are equal to 260lb. at Dantzick? Ans. $224\frac{4}{29}$.

CASE II.

When it is required to find how many of the laſt ſort of coin, weight or meaſure, mentioned in the queſtion, are equal to a given quantity of the firſt.

RULE.

Place the numbers alternately, begining at the left hand, and let the laſt number ſtand on the right hand; then multiply the firſt row for a diviſor, and the ſecond for a dividend.

EXAMPLES.

1. Suppoſe 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouſe, and 100 canes of Thoulouſe=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many ells of Hamburgh are equal to $236\frac{7}{8}$ yards of America?

Ante.		Conſe.		Abridged. Ant.	Con.
100	Amer.	=100	Eng.		
100	Eng.	= 50	Thoul.	5	8
100	Thoul.	=160	Gen.		$236\frac{7}{8}$
100	Gen.	=200	Ham.		
		$236\frac{7}{8}$	Amer.		

$$\frac{236\frac{7}{8}\times8}{5}=379\text{ Ham. Ans.}$$

This needs no further illuſtration. The learner will readily ſee that, this caſe being the reverſe of the former, they are proofs to each other.

2. If 12lb. at Boſton make 10lb. at Amſterdam, and 100lb. at Amſterdam 120lb. at Paris: How many at Paris are equal to 80lb. at Boſton? Ans. 80lb.

3. If 40lb. at Newburyport make 36 at Amſterdam, and 90lb. at Amſterdam make 116 at Dantzick: How many lb. at Dantzick are equal to 244 at Newburyport? Ans. $283\frac{1}{25}$lb.

ARBITRATION OF EXCHANGES.

By this term is underſtood how to chooſe, or determine the beſt way of remitting money abroad with advantage: which is performed by conjoined proportion: Thus,

Suppoſe a merchant has effects at Amſterdam to the amount of 3530 dollars, which he can remit by way of Liſbon at 840 rees per dollar, and thence to Boſton, at 8s. 1d. per milree (or 1000 rees): Or, by way of Nantz, at $5\frac{2}{3}$ livres per dollar, and thence to Boſton at 6s. 8d. per crown; it is required to arbitrate theſe exchanges, that is, to chooſe that which is moſt advantageous?

1 dollar at Amſterdam =840 rees at Liſbon.
1000 rees at Liſbon = 97d. at Boſton.
3530 dollars at Amſterdam.

$$\frac{840\times97\times3530}{1000\times1}=£1198\ 8s.\ 8\tfrac{4}{10}d.$$ by way of Liſbon.

1 dollar at Amſterdam =$5\frac{2}{3}$ livres at Nantz.
6 livres at Nantz =80 pence at Boſton.
3530 dollars at Amſterdam.

$$\frac{5\frac{2}{3}\times80\times3530}{1\times6}=£1059$$ by way of Nantz.

Here it may be obſerved that the difference is £139 8s. $8\frac{4}{10}$d. in favour of remitting by way of Liſbon rather than by Nantz, which depends on the *courſe* of exchange, at that time: but the *courſe* may vary ſo, that, in a ſhort time by way of Nantz, may be better; hence appears the neceſſity and advantage of an extenſive correſpondence, to acquire a thorough knowledge in the *courſes* of exchange, to make this kind of remittance.

FELLOWSHIP.

THE Rules of Fellowſhip are thoſe by which the accompts of ſeveral merchants or other perſons, trading in partnerſhip, are ſo adjuſted, that each may have his ſhare of the gain, or ſuſtain his ſhare of the loſs, in proportion to his ſhare of the joint ſtock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the ſtocks are employed for any certain equal time.

RULE.*

As the whole ſtock is to the whole gain or loſs, ſo is each man's particular ſtock to his particular ſhare of the gain or loſs.

PROOF. Add all the particular ſhares of the gain or loſs together, and, if it be right, the ſum will be equal to the whole gain or loſs.

EXAMPLES.

1. Divide the number 360 into four ſuch parts, which ſhall be to each other, as 3, 4, 5 and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{\begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array}\right\} \text{Anſwer.}$$

360 Proof.

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* That their gain or loſs, in this Rule, is in proportion to their ſtocks, is evident: For, as the times, in which the ſtocks are in trade, are equal, if I put in one half of the whole ſtock, I ought to have one half of the gain; if my part of the ſtock be one fourth, my ſhare of the gain or loſs ought to be one fourth alſo. And generally the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or loſs.

2. B, C, D, and E companied ; B put in 145l. ; C, 219l. ; D, 378l. and E, 417l. with which they gained 569l. : What was the ſhare of each ?

£ s. d. ſhares.

Whole ſtock. Gain.

$$\text{As } 145+219+378+417 : 569 :: \begin{cases} 145 : 71\ \ 3\ \ 8\frac{1}{2}\ \frac{1058}{1159} & \text{B's} \\ 219 : 107\ \ 10\ \ 3\frac{3}{4}\ \frac{375}{1159} & \text{C's} \\ 378 : 185\ \ 11\ \ 6\ \ \frac{552}{1159} & \text{D's} \\ 417 : 204\ \ 14\ \ 5\frac{1}{4}\ \frac{133}{1159} & \text{E's} \end{cases}$$

569 — Proof.

3. K, L, M, and N are concerned in a joint ſtock of D.1000 ; of which K's part is D.150 ; L's D.250 ; M's D.275, and N's 325 ; upon the adjuſtment of their accompts, they have loſt D.337 50c. What is the loſs of each ?
D. c.

Ans. K D.50 62½c. ; L D.84 37½c. ; M D.92 81¼c. ; N D.109 68¾c.

4. F, G and H freighted a ſhip with 68900 feet of boards : F put in 16520 feet ; G 28750 ; and H the reſt ; but in a ſtorm the captain threw overboard 26450 feet : How much muſt each ſuſtain of the loſs ?

Ans. F 6341¼ feet ; G 11036¾, and H 9071½ do.

5. A ſhip, worth D.3000, being loſt at ſea, of which $\frac{1}{8}$ belonged to O, $\frac{1}{2}$ to P, and the reſt to Q : What loſs will each ſuſtain, ſuppoſing D.450 to have been inſured upon her ?

Ans. O's loſs D.312 50c.
P's 937 50
Q's 625

6. X and Y venturing equal ſums of money, cleared by joint trade D.140 ; by agreement, as X did the buſineſs, he was to have 8 per cent. and Y was to have 5 per cent. : What was X allowed for his trouble ?

$$\begin{array}{lllllllll} & \text{D.} & & \text{D.} & & \text{D.} & & \text{D.} & \\ \text{As} & 8+5 & : & 140 & :: & 8 & : & 86\frac{2}{13} & ;\ \text{and,} \\ \text{As} & 8+5 & : & 140 & :: & 5 & : & 53\frac{11}{13} & \end{array}$$

Ans. D.32 30c. $7\frac{9}{13}$m.

7. A bankrupt is indebted to R £120, to S £230, to T £340, and to U £450, and his whole eſtate amounts only to £560 : How muſt it be divided among his creditors ?

Ans. R £58 18 11¼ ; S £112 19 7¼ ; T £167 0 4 ; U £221 1 0½.

8. X, Y, and Z put their money into a joint ſtock ; X put in D.40 ; Y and Z together, D.170 ; they gained .126, of which Y took D.42 : What did X and Z gain, nd Y and Z put in reſpectively ?

As D.210 the whole ſtock : D.126 the whole gain :: .40 X's ſtock : D.24 X's gain.

As D.24 X's gain : D.40 X's ſtock :: D.42 Y's ain : D.70 Y's ſtock. Then D.170—D.70=D.100 Z's tock ; and whole gain D.126—D.66 X's and Y's gain= .60 Z's gain.

9. E, F, G, and H companied ; and gained a ſum of noney, of which E, F, and G took £120 ; F, G, and H, 180 ; G, H, and E, £160 ; and H, E, and F, £140 ; hat diſtinct gain had each ?

The ſum of theſe 4 numbers is £600, and as each nan's money is named 3 times, therefore $\frac{1}{3}$, viz. £200, is he whole gain. Therefore £200—£120 E's, F's, and 's gain=£80 H's gain ; £200—£180 F's, G's, and H's ain=£20 E's gain ; £200—£160 G's, H's and E's gain =£40 F's gain ; and £200—£140 H's, E's and F's gain =£60 G's gain.

DOUBLE FELLOWSHIP,[*]

Or *Fellowſhip with Time*, is occaſioned by the ſhares of artners being continued unequal times.

Rule.

Multiply each man's ſtock, or ſhare, by the time it was ontinued in trade. Then,

As the whole ſum of the products, is to the whole gain r loſs, ſo is each man's particular product, to his particuar ſhare of the gain or loſs.

Examples.

1. A, B, and C hold a paſture in common, for which hey pay 40l. per annum ; A put in 9 oxen for 5 weeks ;

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[*] When times are equal, the ſhares of the gain or loſs are eviently as the ſtocks, as in Single Fellowſhip ; and when the ſtocks re equal, the ſhares are as the times ; wherefore, when neither are qual, the ſhares muſt be as their products.

B, 12 oxen for 7 weeks; and C, 8 oxen for 16 weeks. What muſt each pay of the rent?

9×5=45. 12=7=84, and 8×16=128; then 128+84+45=257.

As 257 : 40 :: 45	As 257 : 40 :: 84	As 257 : 40 :: 128
45	84	40
200	160	257)5120(19
160	320	257
257)1800(7	257)3360(13	2550
1799	257	2313
1	790	237
20	771	20
257)20(0	19	257)4740(18
12	20	257
257)240(0	257)380(1	2170
4	257	2056
257)960(3	123	114
771	12	12
189	257)1476(5	257)1368(5
	1285	1285
	191	83
	4	4
	257)764(2	257)332(1
	514	257
	250	75

2. Four merchants traded in company; L put in 400 dollars for 5 months, M, D.600 for 7 months, N, D.960 for 8 months, and O D.1200 for 9 months; but by misfortunes at ſea, they loſt D.750: What muſt each ſuſtain of the loſs?

Ans. { L D.94 93c. $6\frac{56}{79}$m. N D.227 84c. $8\frac{8}{79}$m.
M 142 40 $5\frac{5}{79}$ O 284 81 $0\frac{10}{79}$

Freightship { M 60 77 [illegible] 233,98, 7 [illegible]

3. A, with a capital of 100l. began trade January 1ſt, 1787, and meeting with ſucceſs in his buſineſs, he took in B as a partner on the firſt day of March following, with a capital of 150l. Three months after that, they admit C as a third partner, who brought into ſtock 180l. and after trading together until the firſt of January, 1788, they found there had been gained ſince A's commencing buſineſs, 177l. 13s. How muſt this be divided among the partners?

$$\text{Ans.}\begin{cases}\text{A } 53\text{l. } 16\text{s. } 8\text{d.}\\ \text{B } 67 \quad 5 \quad 10\\ \text{C } 56 \quad 10 \quad 6\end{cases}$$

4. Two merchants entered into partnerſhip for 18 months; K firſt put into ſtock D.400, and at the end of 8 months he put in 200 more; L, at firſt put in D.1100, and at the end of 4 months took out D.280. Now at the expiration of the time, they found they had gained D.1052: What is each man's juſt ſhare?

Ans. K, D.385 90c. L, D.666 10c.

5. N and O companied; N put in, the 1ſt of January, 150l. but O could not put in any till the 1ſt of May: What did he then put in, to have an equal ſhare with N at the year's end?

$$\begin{matrix} & \text{M.} & & \text{£} & & \text{M.} & & \\ \text{As} & 12 & : & 150 & :: & 8 & : & \dfrac{150\times 12}{8} = \text{£}225 \text{ Ans.}\end{matrix}$$

FELLOWSHIP BY DECIMALS.

Rule.

Divide the whole gain, or loſs, by the whole ſtock, or ſum of all the products, as the caſe requires, and the quotient multiplied ſeverally by each man's ſtock, or product, will give the gain or loſs of each.

Examples.

1. H, I, and K companied; H put in 40l. 5s.; I, 80l. 10s.; and K, 161l. with which they gained 120l.: What is each man's ſhare of the gain?

H's ſtock = 40·25
I's do. = 80·5
K's do. = 161·

Sum total = 281·75)120·000000(·4259+

·4259	·4259	·4259
40·25	80·5	161
21295	21295	4249
8518	34072	25554
17036		4259
	£34·28495	
£17·142475	20	£68·5699
20		20
	5·69900	
2·849500	12	11·3980
12		12
	8·388	
10·194000	4	4 776
4		4
	1·552	
0·776000		3·104

Proof. H's gain 17l. 2s. 10d.+I's gain 34l. 5s. 8¼d.+K's gain 68l. 11s. 4¼d.=119l. 19s. 11d.

2. L, M, and N companied; L put in D 400 for 8 months; M, D.300 for 9 months, and N, D 175 for 12 months; with which they gained D.720. Required the ſhare of each?

D. Mo. Prod.
L 400× 8 = 3200
M 300× 9 = 2700
N 175×12 = 2100

Sum of Products = 8000)720·(·09=quotient.

D.
3200 × ·09 = 288 = L's ſhare,
2700 × ·09 = 243 = M's ſhare, Ans.
2100 × ·09 = 189 = N's ſhare,

3. An inſolvent eſtate, amounting to D.633 60c. is indebted to A D.312 75c. to B D 297, to C D.50 25c. to D 25c. to E D.200, to F D142 50c. and to G D.21 25c.; what proportion will each creditor receive?

$$\frac{633{\cdot}6}{312{\cdot}75+297+50{\cdot}25+{\cdot}25+200+142{\cdot}5+21{\cdot}25}={\cdot}61875\text{; or}$$

61c. 8¾m. on a dollar. And,

$$\left.\begin{array}{r} \text{D. c.} \\ 312{\cdot}75 \\ 297{\cdot} \\ 50{\cdot}25 \\ {\cdot}25 \\ 200{\cdot} \\ 142{\cdot}50 \\ 21{\cdot}25 \end{array}\right\} \times {\cdot}61875 = \left\{\begin{array}{rll} \text{D. c.} & \text{m.} & \\ 193{\cdot}51 & 4\frac{1}{16} & = \text{A's proportion.} \\ 183{\cdot}76 & 8\frac{3}{4} & = \text{B's do.} \\ 31{\cdot}09 & 2\frac{3}{16} & = \text{C's do.} \\ {\cdot}15 & 4\frac{11}{16} & = \text{D's do.} \\ 123{\cdot}75 & & = \text{E's do.} \\ 88{\cdot}17 & 1\frac{7}{8} & = \text{F's do.} \\ 13{\cdot}14 & 8\frac{7}{16} & = \text{G's do.} \end{array}\right.$$

Proof D.633·60

PRACTICE

IS a contraction of the Rule of Three Direct, when the firſt term happens to be an unit, or 1; and has its name from its daily uſe among merchants and tradeſmen, being an eaſy and conciſe method of working moſt queſtions, which occur in trade and buſineſs.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

General Rule.

1. Suppoſe the price of the given quantity to be 1l. or 1s. &c. then will the quantity itſelf be the anſwer at the ſuppoſed price.

2. Divide the given price into aliquot parts, either of the ſuppoſed price, or of one another, and the ſum of the quotients, belonging to each, will be the true anſwer required.

Example.

What is the value of 468 yards, at 2s. 9¼d. per yard?

	£468	s.	d.	Anſwer at £1	s.	d.
2s. 6d. is $\frac{1}{8}$ =	58	10	0	ditto at 0	2	6
3d. is $\frac{1}{10}$ =	5	17	0	ditto at 0	0	3
¼d. is $\frac{1}{12}$ =	0	9	9	ditto at 0	0	0¼
The full price =	£64	16	9	0	2	9¼

In this example it is plain, that the quantity 468 is the anſwer at 1l.; conſequently. as 2s 6d. is $\frac{1}{8}$ of a pound, $\frac{1}{8}$ part of that quantity, or 58l, 10s. is the price at 2s. 6d.;

in like manner, as 3d. is the $\frac{1}{10}$ part of 2s. 6d. ſo $\frac{1}{10}$ part of 58l. 10s. or 5l. 17s. is the anſwer at 3d. ; and as 1qr. is $\frac{1}{12}$ of 3d. ſo $\frac{1}{12}$ of 5l. 17s. or 9s 9d. is the anſwer at 1qr Now as the ſum of all theſe parts is equal to the whole price (2s. 9$\frac{1}{4}$d) ſo the ſum of the anſwers belonging to each price will be the anſwer at the full price required, and the ſame will be true in any example whatever.

GENERAL RULE,

To find the value of goods in Federal Money.—Multiply the price and quantity together ; point off in the product for denominations lower than dollars, as many places as there are in the given price ; or, if there be decimal places in the quantity, (or lower denominations previouſly reduced to decimals) according to multiplication of decimals.

EXAMPLES.

1. What coſt 823 yards, at D.1·29c. per yard ?

D. D.
823×1·29=1061·67 Ans.

2. What coſt 56 yards 2qrs. at D.3·11 per yard ?

56yds 2qrs.=56·5yds. ; and 56·5×3·11=D.175·715 Ans.

Before the queſtions, hereafter given, can be wrought, the following Tables muſt be perfectly gotten by heart.

TABLES.

Aliquot, or even Parts of Money.

Pts. of a s: of a £

d.		s.		£
6	=	$\frac{1}{2}$	=	$\frac{1}{40}$
4	=	$\frac{1}{3}$	=	$\frac{1}{60}$
3	=	$\frac{1}{4}$	=	$\frac{1}{80}$
2	=	$\frac{1}{6}$	=	$\frac{1}{120}$
1$\frac{1}{2}$	=	$\frac{1}{8}$	=	$\frac{1}{160}$
1	=	$\frac{1}{12}$	=	$\frac{1}{240}$
$\frac{3}{4}$	=	$\frac{1}{16}$	=	$\frac{1}{320}$
$\frac{1}{2}$	=	$\frac{1}{24}$	=	$\frac{1}{480}$
$\frac{1}{4}$	=	$\frac{1}{48}$	=	$\frac{1}{960}$

Parts of a £.

s.	d.		£
10	0	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
3	4	=	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
1	8	=	$\frac{1}{12}$
1	4	=	$\frac{1}{15}$
1	3	=	$\frac{1}{16}$
1	0	=	$\frac{1}{20}$
0	10	=	$\frac{1}{24}$
0	8	=	$\frac{1}{30}$
0	5	=	$\frac{1}{48}$
0	2$\frac{1}{2}$	=	$\frac{1}{96}$

Parts of a Dollar

c.		D.
50	=	$\frac{1}{2}$
33$\frac{1}{3}$	=	$\frac{1}{3}$
25	=	$\frac{1}{4}$
20	=	$\frac{1}{5}$
16$\frac{2}{3}$	=	$\frac{1}{6}$
12$\frac{1}{2}$	=	$\frac{1}{8}$
8$\frac{1}{3}$	=	$\frac{1}{12}$
6$\frac{1}{4}$	=	$\frac{1}{16}$
5	=	$\frac{1}{20}$
4	=	$\frac{1}{25}$
2	=	$\frac{1}{50}$

Aliquot, or even Parts of Weight.

Pts. of a Cwt. qrs.	lb.	Cwt.	Pts. of $\frac{1}{2}$C. lb.	$\frac{1}{2}$Cwt	Pts. of $\frac{1}{4}$C. lb.	$\frac{1}{4}$Cwt.	Pts. of a Ton. Cwt.	qr.	T.
2	0	= $\frac{1}{2}$	28	= $\frac{1}{2}$	14	= $\frac{1}{2}$	10	0	= $\frac{1}{2}$
1	0	= $\frac{1}{4}$	14	= $\frac{1}{4}$	7	= $\frac{1}{4}$	5	0	= $\frac{1}{4}$
0	16	= $\frac{1}{7}$	8	= $\frac{1}{7}$	4	= $\frac{1}{7}$	4	0	= $\frac{1}{5}$
0	14	= $\frac{1}{8}$	7	= $\frac{1}{8}$	2	= $\frac{1}{14}$	2	2	= $\frac{1}{8}$
0	8	= $\frac{1}{14}$	4	= $\frac{1}{14}$			2	0	= $\frac{1}{10}$
0	7	= $\frac{1}{16}$					1	1	= $\frac{1}{16}$
0	4	= $\frac{1}{28}$					1	0	= $\frac{1}{20}$

Another Table of aliquot Parts of Money.

Parts of a ſhill.

d.		s.
10	=	$\frac{5}{6}$
9	=	$\frac{3}{4}$
8	=	$\frac{2}{3}$
$7\frac{1}{2}$	=	$\frac{5}{8}$
$4\frac{1}{2}$	=	$\frac{3}{8}$

s.	d.		£.
13	4	=	$\frac{2}{3}$
12	6	=	$\frac{5}{8}$
12	0	=	$\frac{6}{10}$
8	0	=	$\frac{4}{10}$
7	6	=	$\frac{3}{8}$
6	0	=	$\frac{3}{10}$

Parts of a Pound.

s.	d.		£
18	0	=	$\frac{9}{10}$
17	6	=	$\frac{7}{8}$
16	8	=	$\frac{5}{6}$
16	0	=	$\frac{8}{10}$
15	0	=	$\frac{3}{4}$
14	0	=	$\frac{7}{10}$

Parts of a Doll.

c.		D.
$93\frac{3}{4}$	=	$\frac{15}{16}$
$91\frac{2}{3}$	=	$\frac{11}{12}$
90	=	$\frac{9}{10}$
$87\frac{1}{2}$	=	$\frac{7}{8}$
$83\frac{1}{3}$	=	$\frac{5}{6}$
$81\frac{1}{4}$	=	$\frac{13}{16}$

c.		D.
80	=	$\frac{4}{5}$
75	=	$\frac{3}{4}$
70	=	$\frac{7}{10}$
$68\frac{3}{4}$	=	$\frac{11}{16}$
$66\frac{2}{3}$	=	$\frac{2}{3}$
60	=	$\frac{3}{5}$
$58\frac{1}{3}$	=	$\frac{7}{12}$
$56\frac{1}{4}$	=	$\frac{9}{16}$
$43\frac{3}{4}$	=	$\frac{7}{16}$
$41\frac{2}{3}$	=	$\frac{5}{12}$
40	=	$\frac{2}{5}$
$37\frac{1}{2}$	=	$\frac{3}{8}$
$31\frac{1}{4}$	=	$\frac{5}{16}$
30	=	$\frac{3}{10}$
$18\frac{3}{4}$	=	$\frac{3}{16}$

A TABLE OF DISCOUNT PER CENT.

£		s.	d.	
$1\frac{1}{4}$ per cent.	=	0	3	on the pound.
$2\frac{1}{2}$ ——	=	0	6	
$3\frac{3}{4}$ ——	=	0	9	
5 ——	=	1	0	
$6\frac{1}{4}$ ——	=	1	3	
$7\frac{1}{2}$ ——	=	1	6	
$8\frac{3}{4}$ ——	=	1	9	
10 ——	=	2	0	
$12\frac{1}{2}$ ——	=	2	6	
15 ——	=	3	0	

£		s.	d.	
$17\frac{1}{2}$ per cent.	=	3	6	on the pound.
20 ——	=	4	0	
$22\frac{1}{2}$ ——	=	4	6	
25 ——	=	5	0	
30 ——	=	6	0	
35 ——	=	7	0	
40 ——	=	8	0	
45 ——	=	9	0	
50 ——	=	10	0	

CASE I.

When the price of 1yd. lb. *&c. is an even part of one ſhilling*: Find the value of the given quantity at 1s. per yard, lb. &c.; then draw a line underneath and divide by *that even part*, and the quotient will be the anſwer in ſhillings, which muſt always be brought into pounds.

EXAMPLES.

1. What will 354½yds. coſt at, ¼d. per yard?

| ¼d. | $\frac{1}{48}$ | 354 6 value of 354½yds. at 1s. per yard.

Ans. £0 7 4½ value of 354½yds. at ¼d. per yard.

Or thus.
£ s. d. s. d.
8)17 14 6=354 6
6)2 4 3¾
7 4½ Ans. as before.

Or divide by 8 and 6, thus,
8)354 6
6)44 3¾
7 4½ Ans.

2. What will 759¾ yards come to, at 3d. per yard?

3d. | ¼ | 759 9 value at 1s. per yard.
2|0)18|9 11¼
Ans. £9 9 11¼ value at 3d. per yard.

£ s. d.
Or thus. | 3d. | ¼ | 37 19 9 value at 1s. per yd.
Ans. £9 9 11¼ value of 759 yards, at 3d. per yard.

Queſtions. yds.		Anſwers. £	s.	d.
3. 642	at ¼d. per yard.	0	13	4½
4. 567½	— 1½d. ——	3	10	11¼
5. 475¼	— 4d. ——	7	18	5

CASE II.

When the price is pence, and no even part of a ſhilling: Find the value of the given quantity at 1s. per yard divide the pence into aliquot parts, for diviſors, and the ſum of the quotients ariſing from them will be the anſwer.

EXAMPLES.

1. What will 487½ yards come to, at 5d. per yard?

3d.	¼	24	7	6	value of 487½yds. at 1s. per yard.
2d.	⅙	6	1	10½	value of ditto at 3d. per yard.
		4	1	3	value of ditto at 2d. per yard.
	Ans.	£10	3	1½	value of ditto at 5d. per yard.

3. 912½ yards, at 9d. per yard. Ans. 34l. 4s. 4½d.
4. 745¾ yards, at 11d. per yard. Ans. 34l. 3s. 7¼d.

CASE III.

When the price is between one and two ſhillings: Find the value of the quantity at 1s. per yard, &c. which value being divided by *thoſe even parts,* which the pence are of 1s. and the quotient or quotients ariſing therefrom, added thereto, the ſum will be the anſwer.

EXAMPLES.

1. What will 758½ yards come to, at 1s. 9d. per yard?

6d.	½	37	18	6	value at 1s. per yard.
3d.	¼	18	19	3	value at 6d per yard.
		9	9	7½	value at 3d. per yard.

Ans. £66 7 4½ value of 758½ yards, at 1s. 9d. per yard.

Queſtions.			Anſwers.		
2. 793 yards, at	12¾d.		£42	2	6¾
3. 847½ ——	1s.	1d.	45	18	1½
4. 647¾ ——	1s.	5d.	45	17	7¾
5. 752½ ——	1s.	10d.	68	19	7

CASE IV.

When the price is an even part of a pound: Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide by *that part*; the quotient will be the anſwer.

EXAMPLES.

1. What will 156¾ yards come to, at 3s. 4d. per yard:

s. d.		£ s. d.	
3 4	⅙	156 15 0	price at 1l. per yard.
		£26 2 6	Ans. price at 3s. 4d. per yard.

Queſtions. Yds.		s. d.		Anſwers. £ s. d.
2. 516¾	at	1 0	per yard.	25 16 9
3. 624	—	1 3	——	39 0 0
4. 719½	—	1 4	——	47 19 4
5. 648	—	1 8	——	54 0 0

CASE V.

When the price wants an even part of a pound: Firſt find the value of the given quantity at 1l. per yard, &c. then divide it by *that even part* which is wanting, and ſubtract this quotient therefrom; the remainder will be the anſwer.

EXAMPLES.

1. What will 167½yds. coſt, at 17s. 6d. per yard?

s. d.		£ s. d.	
2 6	⅛	167 10 0	value at 1l. per yard.
		20 18 9	ditto at 2s. 6d. per yard.
		£146 11 3	Ans. value at 17s. 6d. per yard.

Queſtions. Yds.		s. d.		Anſwers. £ s. d.
2. 347½	at	13 4	per yd.	231 13 4
3. 485¾	—	15 0	——	364 6 3
4. 614	—	16 0	——	491 4 0
5. 912¼	—	17 6	——	798 4 4½

CASE VI.

When the price is ſhillings, pence, and farthings, and not an even part of a pound: Multiply the given quantity by the ſhillings in the price of 1 yard, &c and take parts of parts from the quantity for the pence, &c; then add them together, and their ſum will be the anſwer in ſhillings, &c. Or you may let the given quantity ſtand as pounds per yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their ſum will be the anſwer.

N. B. I adviſe the learner to work the following examples both ways, by which means he will be able to diſcover the moſt conciſe method of performing ſuch queſtions in buſineſs, as may fall under this caſe.

EXAMPLES.

1. What will 248½ yards coſt, at 7s. 6d. per yard?

6	½	248s.	6d.	value of 248½ yards, at 1s. per yard.
			7	
		1739	6	value of ditto at 7s. per yard.
		124	3	value of ditto at 6d. per yard.
		2|0)186|3	9	
Ans.		£93 3	9	value of ditto at 7s. 6d. per yard.

Or thus:

		£	s.	d.	
6	½	12	8	6	value of 248½ yards, at 1s. per yd.
Mult. by				7	
		86	19	6	value of ditto at 7s. per yard.
		6	4	3	value of ditto at 6d. per yard.
Ans.		£93	3	9	

By the latter part of this caſe.

5 0	¼	248	10	0	value of 248½ yards, at 1l. per yd.
2 6	½	62	2	6	value of ditto at 5s. per yard.
		31	1	3	value of ditto at 2s. 6d. per yard.
Ans.		£93	3	9	value of ditto at 7s. 6d. per yard.

Queſtions.						Anſwers.		
	Yds.		s.	d.		£	s.	d.
2.	68½	at	4	6	per yd.	15	8	3
3.	124	—	5	8	——	35	2	8
4.	146	—	14	9	——	107	13	6
5.	218½	—	12	6	——	136	11	3

CASE VII.

When the price of the yard, lb. *&c. is pounds, ſhillings, and pence:* Firſt, multiply the quantity by the pounds, and if the ſhillings and pence be an even part of a pound, divide the given quantity by *that part*, and add the quotient to the product for the anſwer. But if they be not an even part of a pound, you muſt take parts of parts, and add them together as before. Or, reduce the pounds and ſhillings into ſhillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their ſum will be the anſwer in ſhillings, &c.

N. B. The learner ſhould work the following queſtions both ways.

Examples.

1. What will 156 yards come to, at 3l. 6s. 8d. per yd.?

s. d.
| 6 8 | $\frac{1}{3}$ | 156 0 0 value at 1l. per yard.
3
———
468 0 0
52
———
Ans. £520 0 0

Or thus:

| 4 | $\frac{1}{3}$ | 156 value at 1s. per yard.
| 4 | $\frac{1}{3}$ | 66 ſhillings in the price of 1 yard.
———
936
936
———
10296 value at 3l. 6s. per yard.
52
52
———
2|0)1040|0
———
£520 0 0

	Queſtions. Yds.		£	s.	d.		Anſwers. £	s.	d.
2.	345½	at	6	5	0	per yd.	2159	7	6
3.	59¾	—	3	6	8	—	199	3	4
4.	75	—	5	3	4	—	387	10	0
5.	68	—	4	6	0	—	292	8	0

CASE VIII.

When the price of one hundred weight, &c. is of ſeveral denominations, and the quantity likewiſe: Multiply the price by the integers, and take parts for the reſt from the price of an integer; which, added together, will be the anſwer.

EXAMPLES.

1. What will 9cwt. 3qrs. 14lb. of ſugar come to, at 4l. 17s. 4d. per cwt.?

qrs.	lb.		£	s.	d.		£	s.	d.
2	0	½	4	17	4	price of 1cwt.			
1	0	½			9				
0	14	½							
			43	16	0	price of	9	0	0
			2	8	8	—	0	2	0
			1	4	4	—	0	1	0
			0	12	2	—	0	0	14
		Ans.	£48	1	2	price of	9	3	14

	Queſtions. cwt.	qr.	lb.		£	s.	d.		Anſwers. £	s.	d.
2.	8	1	16	at	5	17	9	per cwt.	49	8	2¾
3.	7	3	19	—	7	12	8	—	60	9	0½
4.	12	1	24	—	3	18	10	—	49	2	7¼
5.	16	2	17	—	2	15	11	—	46	11	1

CASE IX.

When the price is at any of the rates in the ſecond Practice Table of aliquot parts: Multiply the given quantity by the numerator, and divide that product by the denominator; if the price be pence, the quotient will be the anſwer in ſhillings; if ſhillings, the anſwer will be pounds.

Examples.

1. What will 379 yards, at 4½d. per yard, come to?

```
   379
     3
   ----
8)1137
   ----
2|0)14|2  1½
   ----
Ans. £7  2  1½
```

2. What will 149 yards, at 6s. per yard, come to?

```
    149
      3
   ----
1|0)44|7
   ----
Ans. £44 14
```

Queſtions.			s.	d.		Anſwers. £	s.	d.
3.	127	yards, at	0	7½	per yard.	3	19	4½
4.	249	ditto	7	6	——	93	7	6
5.	357	ditto	12	6	——	223	2	6

CASE X.

To find the value of goods ſold by particular quantities: viz. I. By the ſcore. II. Round timber. III. By 5 ſcore to the hundred. IV. By 112 to the hundred. V. By 6 ſcore to the hundred. VI. By the great groſs. VII. By the thouſand.

I. *To find the value of Goods ſold by the Score.*

The price of 1 is given, to find the price of 1 ſcore.

If the given price be ſhillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the anſwer in pounds, and the remainder will be ſo many times 1s. 8d.

Examples.

1. At 9d. each, what is that per ſcore?

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12)9d.(·75=£0 15 0 Ans.
Or by inverting the queſtion.
1 ſcore=20=1s. 8d.
               9
            ----
          15s. 0
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2. At 4s. 9d. each, what is that per ſcore?

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    4s. 9d.
    12
    --
12)57d.
   ----
£4 15s. Ans.
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It may be remarked, that when the price is ſhillings and pence, the anſwer will be juſt ſo many pounds as there are ſhillings, and ſo many times 1s. 8d. as there are pence. If farthings are given, for 1qr. reckon 5d. for 2qrs. 10d. and for 3qrs. 1s. 3d.

TABLE of Aliquot Parts. 20 the Integer.

2 = $\frac{1}{10}$	6 = $\frac{3}{10}$	12 = $\frac{6}{10}$	16 = $\frac{8}{10}$
4 = $\frac{1}{5}$	8 = $\frac{4}{10}$	14 = $\frac{7}{10}$	18 = $\frac{9}{10}$
5 = $\frac{1}{4}$	10 = $\frac{1}{2}$	15 = $\frac{3}{4}$	

3. What coſt 7, at 2s. 9d. per ſcore ?

		s.	d.
5	$\frac{1}{4}$	2	9
2	$\frac{1}{10}$		
		0	$8\frac{1}{4}$
		0	$3\frac{1}{4}$
7	=	0	$11\frac{1}{2}$

4. What coſt 17, at 19s. 10d. per ſcore ?

		s.	d.
10	$\frac{1}{2}$	19	10
5	$\frac{1}{4}$		
2	$\frac{1}{10}$	9	11
		4	$11\frac{1}{2}$
		1	$11\frac{3}{4}$
16	=	16	$10\frac{1}{4}$

II. *Round Timber.*

Forty feet make a load or ton of round timber.

If the given price of a foot be ſhillings,

Rule.

Multiply the given price by 2, and the product will be the anſwer in pounds.

5. What coſt a ton, at 3s. per foot ? 3s×2=6l. Ans.
6. What coſt a ton, at 9s. per foot ? 9s.×2=18l. Ans.

If the given price of 1 foot be pence only, or ſhillings and pence, divide the given price, in pence, by 6. The quotient will be the anſwer in pounds, and the remainder will be ſo many times 3s. 4d.

7. What coſt 40 feet, at 17d. per foot ?

6)17

£2 16 8 Ans.

8. At 1s. 9d. per foot, what coſt a ton ?

6)21

£3 10 Ans.

If the given price be farthings only, or pence and farthings, divide the given price, in farthings, by 6 ; then divide *that quotient* by 4, and *this laſt quotient* will be the anſwer.

9. At 3qrs. per foot, what coſt a ton?

6)3

4)0 10

£0 2 6 Ans.

10. At 13¼d. per foot, what coſt a ton?

13¼

4

6)53

4)8 16 8

£2 4 2 Ans.

Or, Suppoſe every ſhilling in the price to be 2l. every penny to be 3s. 4d. and every farthing to be 10d.

11. What coſt 40 feet, at 3qrs. per foot?

3qrs.×10=2s. 6d. Ans.

12. What coſt 40, at 15½d. per foot?

s. d.
1 0× 2=£2 0 0
3 4× 3= 0 10 0
0 ½×10= 0 1 8

£2 11 8

III.* *To find the value of Goods ſold by 5 ſcore to the Hundred.*

1ſt. If the given price be pounds and ſhillings, or ſhillings only,

RULE.

Multiply the given price, in ſhillings, by 5, and the product will be the anſwer in pounds.

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* *In Federal Money.*—Remove the decimal point two places to the right for the anſwer.

EXAMPLES.

1. What coſt 100 yards, at D.2 50c. per yard?
D.2·50×100=D.250, Ans.
2. What coſt 100 yards, at 75c. per yard?
D.75×100=D.75, Ans.
3. What coſt 100 yards, at 5c. 6¼m. per yard?
D.05625×100=D.5·625, Ans.
4. What coſt 100 yards, at 37c. 5m. per yard?
Ans. 37dols. 50c.
5. What coſt 100 yards, at 68c. 7½m. per yard?
Ans. 68dols. 75c.

13. At 19s. per yard, what coſt 100 yards?

19s.
5
£95 Ans.

14. At 4l. 13s. per cwt. what coſt 100 cwt. or 5 tons?

4 13
20
93
5
£465 Ans.

2d. If the given price of 1 be pence only, or ſhillings and pence,

RULE.

Multiply the given price, in pence, by 5; then divide that product by 12. The quotient will be pounds; and the remainder ſo many times 1s. 8d.

15. If 1 yard coſt 9d. what coſt 100 yards?

9
5
12)45
£3 15 Ans.

16. What coſt 100 buſhels, at 35s. 4d. per buſhel?

s. d.
35 4
12
424
5
12)2120
£176 13 4 Ans.

Or,

35s. 4d.
| 4d. | $\frac{1}{3}$ | 5
175
1 13 4
£176 13 4

Here 5 is divided by $\frac{1}{3}$.

3d. If the given price of 1 be ſhillings and pence: Multiply the price by 5, and the product under the place of ſhillings will be the anſwer in pounds, and the product under the place of pence, will be ſo many times 1s. 8d.

17. At 2s. 5d. per buſhel, what coſt 100 buſhels?

s. d.
2 5
5
—
12 1
—
£12 1 8 Ans.

18. At 25s. 3d. per ton, what coſt 100 tons?

s. d.
25 3 1s. 8d.×3=5s.
5
—
126 3
—
£126 5 Ans.

4th.* To find the price of 1, at ſo much per hundred of 5 ſcore.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the anſwer in pence.

But if the price be pounds only:

RULE.

Divide the given price by 5, and the quotient will be the anſwer in ſhillings.

19. If 100yds. coſt 65l. what coſt 1 yard?

5)65
—
13s. Ans.

20. If 100yds. coſt 2l. 18s. 4d what is that per yard?

£ s. d.
2 18 4
12
—
5)35 0 0
—
7d. Anſwer.
—

21. If 100 yards coſt 11l. 7s. 9d. what coſt 1 yard?

£ s. d.
11 7 9
12
—
5)136 13 0
—
12)27 6 7
—
2s 3¼d. Ans.

In dividing 27 by 12 (in the 21ſt queſtion) the quotient is 2s and the remainder 3d the 6 is $\frac{6}{20}$ of a penny, = one farthing, and the 7 is of no account.

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* *In Federal Money.*—Remove the decimal point two places to the left for the anſwer.

EXAMPLES.

1. If 100 yards coſt D.250, what coſt 1 yard?
D.250÷100=D 2 50 Ans.

2. If 100 yards coſt D 75, what coſt 1 yard?
D.75÷100=D.75, Ans.

TABLE of Aliquot Parts. 100 the Integer.

5 =	$\frac{1}{20}$	25 =	$\frac{1}{4}$	50 =	$\frac{1}{2}$	75 =	$\frac{3}{4}$
10 =	$\frac{1}{10}$	30 =	$\frac{3}{10}$	60 =	$\frac{6}{10}$	80 =	$\frac{8}{10}$
20 =	$\frac{1}{5}$	40 =	$\frac{4}{10}$	70 =	$\frac{7}{10}$	90 =	$\frac{9}{10}$

22.* At 3l. 7s. 6d. per 100, what will 23 coſt?

$$\begin{array}{r|c|rrr l} & & £ & s. & d. & \\ 20 & \frac{1}{5} & 3 & 7 & 6 & \\ \hline & & 0 & 13 & 6 & \\ 2 & \frac{1}{10} & 0 & 1 & 4 & \text{Add.} \\ 1 & \frac{1}{2} & 0 & 0 & 8 & \\ \hline 23 & = & £0 & 15 & 6 & \text{Anſwer.} \end{array}$$

23. At 2l. 1s. 10d. per 100, what coſt 18?

$$\begin{array}{r|c|rrr l} & & £ & s. & d. & \\ 20 & \frac{1}{5} & 2 & 1 & 10 & \\ \hline & & 0 & 8 & 4\frac{2}{5} & \text{Sub.} \\ 2 & \frac{1}{10} & 0 & 0 & 10 & \\ \hline 18 & = & £0 & 7 & 6\frac{2}{5} & \text{Ans.} \end{array}$$

24. At 5l. 9s. 6d. per 100, what coſt 35?

$$\begin{array}{r|c|rrr l} & & £ & s. & d. & \\ & & 5 & 9 & 6 & \\ & & & & 3 & \\ \hline & 10) & 16 & 8 & 6 & \\ \hline 5 & \frac{1}{6} & 1 & 12 & 10 & \text{Add} \\ & & & 5 & 5\frac{2}{3} & \\ \hline & & £1 & 18 & 3\frac{2}{3} & \text{Ans.} \end{array}$$

3. If 100 yards coſt D.5 62c. 5m. what coſt 1 yard?
D.5·625÷100=D.·05625=5c. 6¼m. Ans.

4. If 100 yards coſt D.37 50c. what coſt 1 yard? Ans. 37c. 5m.

5. If 100 yards coſt D 68 75c. what coſt 1 yard? Ans. 68c. 7½m.

* *To find the value of any number at a given price per* 100 *in Federal Money*—Multiply the price per 100 by the given quantity, and point off two right hand figures, in the product, more than required by multiplication of decimals. Or, point off the two right hand places in the given quantity, and multiply and point, as in multiplication of decimals.

EXAMPLES.

1. What coſt 56 yards, at D.87 50c. per 100 yards?

$$\frac{D.87\cdot5 \times 56}{100} = D.49 \text{ Ans. Or, } D.87\cdot5 \times \cdot56 = D.49 \text{ as before.}$$

2. What coſt 45½lb. of beef, at D.5½ per 100?

$$\frac{D.5\cdot5 \times 45\cdot5}{100} = D.2\cdot5025 = D.2\ 50c.\ 2\tfrac{1}{2}m. \text{ Ans.}$$

Or, D.5·5×·455lb.=D.2·5025, as before.

IV. *To find the value of Goods ſold by 112lb. the Cwt.*

The price of 1lb. is given to find the value of 1cwt.

Rule.

For a farthing, account 2s. 4d. per cwt. For half a penny, 4s. 8d. For 3qrs. 7s. And for every penny 9s. 4d. per cwt.

25. What coſt 1cwt. at 3½d. per cwt.?

At 1d. per lb. s. d.
1cwt. coſts 9 4
3

At 3d. £1 8 0 } Add
At ½d. 0 4 8 }

£1 12 8 Ans.

26. At 8¾d. per lb. what coſt 1cwt.

At 1d. per lb. s. d.
1cwt. coſts 9 4
8

At 8d. £3 14 8 } Add
At ¾. 0 7 0 }

£4 1 8 Ans.

V. *To find the value of Goods ſold by 6 Score to the Hundred.*

The price of 1 is given, to find the price of 1 hundred.

Rule.

Suppoſe every penny in the price to be ſo many pounds, and for every farthing, ſuch a part of a pound, as they are of a penny; then, half of that ſum will be the anſwer.

27. At 4½d. per yard, what coſt 120 yards?

£ s.
2)4 10

£2 5 Ans.

28. At 16s. 9¼d. per yard, what coſt 120 yards?

s. d.
16 9¼
12

2)201l. 5s.

£100 12 6 Ans.

3. What coſt 375 yards, at 375 dollars per 100 yards?
Ans. D.1406 25c.

4. What coſt 54 yards, at 16 dollars per 100 yards?
Ans. D.8 64c.

5. What coſt 512 yards, at 6 dollars 25 cents per 100 yards?
Ans. D.32.

To find the price of 1, at ſo much per hundred of 6 ſcore.

Rule.

Multiply the price by 2, then call the pounds ſo many pence, and the ſhillings ſuch a part of a penny, as they are of a pound, and you will have the anſwer.

39. If 120 yards coſt 3l. 1s. what coſt 1 yard?

£ s.
3 12
2
7 4

Ans. 7$\frac{1}{3}$d.

30. If 120 yards coſt 5l. 18s. 6d. what coſt 1 yard?

£ s. d.
5 18 6
2
11 17 0

11$\frac{3}{4}$d. + $\frac{2}{3}$qr. Ans.

TABLE of Aliquot Parts. 120 the Integer.

Alſo,

6 = $\frac{1}{20}$	24 = $\frac{1}{5}$	36 = $\frac{3}{10}$	72 = $\frac{3}{5}$	96 = $\frac{4}{5}$
10 = $\frac{1}{12}$	30 = $\frac{1}{4}$	45 = $\frac{3}{8}$	75 = $\frac{5}{8}$	100 = $\frac{5}{6}$
12 = $\frac{1}{10}$	40 = $\frac{1}{3}$	48 = $\frac{2}{5}$	80 = $\frac{2}{3}$	105 = $\frac{7}{8}$
15 = $\frac{1}{8}$	60 = $\frac{1}{2}$	50 = $\frac{5}{12}$	84 = $\frac{7}{10}$	108 = $\frac{9}{10}$
20 = $\frac{1}{6}$		70 = $\frac{7}{12}$	90 = $\frac{3}{4}$	110 = $\frac{11}{12}$

31. At 3l. 17s. 6d. per hundred, what coſt 14?

		£ s. d.
12	$\frac{1}{10}$	3 17 6
		0 7 9
2	$\frac{1}{6}$	0 1 3$\frac{1}{2}$

} Add.

14 = £0 9 0$\frac{1}{2}$ Ans.

32. At 2l. 13s. 6$\frac{1}{2}$d. per hundred, what coſt 49?

		£ s. d.
40	$\frac{1}{3}$	2 13 6$\frac{1}{2}$
8	$\frac{1}{5}$	0 17 10 0$\frac{2}{3}$
1	$\frac{1}{8}$	0 3 6 3$\frac{1}{8}$
		0 0 5 1

49 =£1 1 10 1 Ans.

33. At 1l. 19s. 3d. per hundred, what coſt 75?

£ s. d.
1 19 3
5
8)9 16 3
£1 4 6$\frac{1}{4}$

VI. *To find the value of Goods ſold by the Great Groſs.*

Note. 12 make 1 dozen, 12 dozen 1 ſmall groſs, 12 ſmall groſs 1 great groſs.

The price of 1 dozen being given, in pence to find the price of a great groſs.

Rule.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the anſwer in pounds, &c.

For proof do the contrary.

N. B. If the price of 1 be given, the price of one ſmall groſs is found after the ſame manner.

34. What coſt 1 great groſs, at 18d. per dozen?

3

5)54

£10 16.

35. At 4s. 3d. per doz. what coſt 1 great groſs?

4s. 3d.	Or,	Or,
12	s. d.	£ s.
	4 3	144=7 4
51d.	12	4
3		
	2 11 0	3d. | $\frac{1}{4}$ | 28 16 } Add
5)153	12	1 16 }
£30 12 Ans.	£30 12 0	£30 12

TABLE of Aliquot Parts. 144 the Integer.

		Alſo,		
12 = $\frac{1}{12}$	36 = $\frac{1}{4}$	32 = $\frac{2}{9}$	84 = $\frac{7}{12}$	128 = $\frac{8}{9}$
16 = $\frac{1}{9}$	48 = $\frac{1}{3}$	60 = $\frac{5}{12}$	96 = $\frac{2}{3}$	132 = $\frac{11}{12}$.
18 = $\frac{1}{8}$	72 = $\frac{1}{2}$	64 = $\frac{4}{9}$	108 = $\frac{3}{4}$	
24 = $\frac{1}{6}$		80 = $\frac{5}{9}$	120 = $\frac{5}{6}$	

36. At 2l. 12s. 9d. per great groſs, what coſt 45 dozen?

		£ s. d.
36	$\frac{1}{4}$	2 12 9
		0 13 2¼ } Add.
9	$\frac{1}{4}$	0 3 3½ }
45		= £0 16 5¾ Ans.

37. What coſt 117 doz. en, at 9l. 13s. 7d. per great groſs?

£ s. d.
9 13 7
3

Doz.
108 | ¼ | 29 0 9
9 | $\frac{1}{12}$ | 7 5 2¼ } Add
12 1 }

117 = £7 17 3¼ Ans.

38. At 3l. 16s. 8d. per great groſs, what coſt 7 great groſs and 96 dozen?

£ s. d.
3 16 8
2

3)7 13 4
Doz.
96= 2 11 1¼
Top line ×7=26 16 8

£29 7 9¼

VII.* *To find the value of Goods ſold by the Thouſand.*

The price of 1 is given to find the price of 1000.

RULE.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the anſwer in pounds, &c.

39. At 6d. each, what coſt 1000?

6
50
12)300
£25 Ans.

Or, as 1000s. are 50l. take parts for the pence out of 50.

| 6d. | ½ | 50
Ans. 25

40. What coſt 1000, at 2¼d. each?

2d. | ⅙ | 50
¼ | ⅛ | 8 6 8 } +
1 2 10 }

£9 7 6 Ans.

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* *In Federal Money.*—Remove the decimal point three places to the right or left, as the caſe requires, for the anſwer.

EXAMPLES.

1. What coſt 5000 yards, at 5 cents per yard?

·05×1000=050=D.50, Ans.

VIII.* *To find the price of one at ſo much per thouſand.*

Rule.

Multiply the price by 12; divide the product by 50; then take the pounds for ſo many pence, and the ſhillings for ſuch a part of a penny as they are of a pound, which will be the anſwer.

At 5l. 4s. 2d. per 1000, what coſt 1?

£	s.	d.
5	4	2
		12

$$50\left\{\begin{array}{l} 5)\,62\ \ 10\ \ 0 \\ 10)\,12\ \ 10 \end{array}\right.$$

£1 5

Ans. $1\frac{1}{4}$d.

42. At 354l. 3s. 4d. per 1000, what coſt 1?

£	s.	d.
354	3	4
		12

$$50\left\{\begin{array}{l} 10)\,4250\ \ 0\ \ 0 \\ 5)\,425 \end{array}\right.$$

85

Ans. 7s. 1d.

Or,

		£	s.	d.
100	$\frac{1}{10}$	354	3	4
10	$\frac{1}{10}$	35	8	4
1	$\frac{1}{10}$	3	10	10
	Ans.	0	7	1

TABLE of Aliquot Parts. 1000 the Integer.

	Alſo,	
$50 = \frac{1}{20}$		
$100 = \frac{1}{10}$	$300 = \frac{3}{10}$	$700 = \frac{7}{10}$
$125 = \frac{1}{8}$	$375 = \frac{3}{8}$	$750 = \frac{3}{4}$
$200 = \frac{1}{5}$	$400 = \frac{2}{5}$	$800 = \frac{4}{5}$
$250 = \frac{1}{4}$	$600 = \frac{3}{5}$	$875 = \frac{7}{8}$
$500 = \frac{1}{2}$	$625 = \frac{5}{8}$	$900 = \frac{9}{10}$

2. What coſt 1000 yards, at 12 cents 5 mills per yard?

Ans. D.125.

3. If 1000 yards coſt D.37 50c what coſt 1 yard?

D.37·5÷1000=D.·0375=3c. $7\frac{1}{2}$m. or $3\frac{3}{4}$c. Ans.

4. If 1000 yards coſt D.1625, what coſt 1 yard?

Ans. D.1 62c. 5m.

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* See the preceding Note.

[illegible] per [illegible], what cost 115?

[illegible] Ans.

[illegible] Ans.

[illegible] per [illegible], what cost 875?

[illegible] Ans.

[illegible] per 1000?

[illegible] Ans.

[illegible] Ans.

[illegible] at a given price per thousand, is [illegible] the price per [illegible] the given quantity, [illegible] figures in the product more than re[illegible] decimals. Or, point off the three right [illegible] quantity; and multiply and point as in [illegible] decimals.

EXAMPLES.

[illegible] per 1000.

[illegible] =D.11 57c. 5m. Ans.

[illegible] at D.16 per 1000?

Ans. D.626 80c.

[illegible] cents per 1000?

Ans. 48c. 7½m. or, 48¾c.

CASES IN FEDERAL MONEY.

CASE I.

When the price of 1 *is an aliquot part of a dollar.*—Divide the quantity by the denominator of the fraction, which the price is of a dollar, and the quotient will be the anſwer in dollars, &c.

EXAMPLES.

1. What coſt 227 yards, at 50 cents per yard?

c.	D	D.
50	$\frac{1}{2}$	227=price at D.1 per yard.
		D 113 50c. Ans.

2. What coſt 265 yards, at 12c. 5m. per yard?
Ans. D 33 12c. 5m.

3. What coſt $269\frac{1}{2}$ yards, at $16\frac{2}{3}$c. per yard?

$16\frac{2}{3}$c.	$\frac{1}{6}$	269·5 price at D.1.
		D.44 91c. 7m. Ans.

4. What coſt 1050 yards, at $6\frac{1}{4}$c. per yard?
Ans. D 65 62c. 5m.

CASE II.

When the price of 1 *is two or more aliquot parts of a dollar added together:* Divide the given number firſt for one aliquot part, then for another, &c. the quotients added together, will be the anſwer.

EXAMPLES.

1. What coſt 298 yards, at 75 cents per yard?

c.	D.	D.		
50	$\frac{1}{2}$	298	price at D.1	
25	$\frac{1}{4}$	149	ditto at	·50
		74 50	ditto at	·25
	Ans.	223 50	ditto at	·75

2. What coſt 927 yards, at $53\frac{1}{3}$c. per yard?
Ans. D.494 40c.

3. What coſt 618 yards, at $87\frac{1}{2}$c. per yard?
Ans. D.540 75c.

4. What coſt 328 yards, at 57c. 5m. per yard?
Ans. D.188 60c.

CASE III.

When the price of 1 is the difference between two aliquot parts of a dollar: Find the price at the greater aliquot part, and then at the leſs, and their difference will be the anſwer.

Examples.

1. What coſt 328 yards, at 13⅓ cents per yard?

c.	D.	D.		
33⅓	⅓	328	price at D.1	
20	⅕	109 33⅓	ditto at	·33⅓c.
		65 60	ditto at	·20
	Ans.	43 73⅓	ditto at	13⅓c.

2. What coſt 817 yards, at 30c. per yard?
Ans D 245 10c.

3. What coſt 296 yards, at 15 cents per yard?
Ans D.44 40c.

CASE IV.

When the price of 1 is any ſum leſs than a dollar: Divide the given price into aliquot parts, either of a dollar, or of each other: find the price at each, and add them together for the anſwer.

Examples.

1. What coſt 279 yards, at 31 cents per yard?

25c.	¼D.	279·	price at D.1	
5	⅕ of 25c.	69 75	ditto at	25c.
1	⅕ of 5	13·95	ditto at	5
		2·79	ditto at	1
	Ans.	D 86·49	ditto at	31c.

2. What coſt 953 yards, at 57c. per yard?
Ans. D.543 21c.

3. What coſt 839 yards, at 36c. per yard?
Ans. D.302 4c.

CASE V.

When the price of 1 is any ſum between 1 and 2 dollars: The quantity itſelf in dollars, is the price at D.1; then finding, by the preceding rules, the price at the parts of D. 1, the ſum of the whole is the anſwer.

EXAMPLES.

1. What coſt 386 yards, at D. 1·65c. per yard?

50c.	½D.	386	price at D. 1	
10	⅕ of 50c.	193	ditto at	50c.
5	½ of 10	38 60	ditto at	10
		19 30	ditto at	5
		Ans. D.636 90	ditto at D. 1	65c.

2. What coſt 849 yards, at D. 1·72 per yard?
Ans. D. 1460·28.

3. What coſt 294 yards, at D. 1·18 per yard?
Ans. D.346·92.

CASE VI.

When the price of 1 *is any number of dollars and parts of a dollar:* Multiply the quantity by the number of dollars; and, finding, by the preceding rules, the price at the parts of D. 1, the ſum of the whole is the anſwer.

EXAMPLES.

1. What coſt 395 yards, at D 3 24c. per yard?

c.		D.	c.			
20	⅕D.	395		price at D. 1		
		3				
4	⅕ of 20c.	1185		ditto at	3	
		79		ditto at		20c.
		15	80	ditto at		4
		Ans. D.1279	80	ditto at D.	3	24c.

2. What coſt 269 yards at D. 2 60c. per yard?
Ans. D.699 40c.

3. What coſt 694 yards, at D. 12·10 per yard?
Ans. D.8397 40c.

4. What coſt 318 yards, at D. 4·125 per yard?
Ans D.1311 75c.

5. What coſt 175 yards, at D4·44 per yard?
Ans. D.777.

CASE VII.

When the price of 1 *contains the ſame aliquot part of a dollar any number of times exactly ; or, in other words, when the price of* 1 *has an aliquot part, which is alſo an aliquot part of a dollar :* Firſt, find the value of the given quantity at the aliquot part ; then multiply this by the number of times, which the aliquot part is contained in the given ſum, for the anſwer.

Or, Since the price in this caſe is always ſuch a number, as, being divided by the aliquot part, will make the numerator of a fraction, of which the denominator is the denominator of that fraction, which the aliquot part is of a dollar : Multiply the quantity by the numerator, and divide the product by the denominator, (or, when convenient, divide the quantity by the denominator, and multiply the quotient by the numerator,) for the anſwer.

EXAMPLES.

1. What coſt 384 yards, at $87\frac{1}{2}$ cents per yard ?

$12\frac{1}{2}$c.=$\frac{1}{7}$ of ·875=| $\frac{1}{8}$D. | 384·=price at D.1

48·=ditto at ·$12\frac{1}{2}$
× 7 × 7

Ans. D.336·=ditto at ·$87\frac{1}{2}$

Or thus,

·875=D.$\frac{7}{8}$, and $384\times\frac{7}{8}=\frac{384\times7}{8}\left(=\frac{384}{8}\times7\right)$=D.336 anſwer as before.

2. What coſt 842 yards, at $66\frac{2}{3}$c. per yard ?
Ans. D.561 $33\frac{1}{3}$c.

3. What coſt 912 yards, at 55 cents per yard ?
Ans. D.501 60c.

MISCELLANEOUS QUESTIONS.

1. What coſt 300 yards, at 27 cents per yard ?
Ans. D.81.

2. What coſt 917 yards, at D.1·125 per yard ?
Ans. D.1031 62c. 5m.

3. What coſt $35\frac{1}{5}$ yards, at 35 cents per yard ?
Ans. D.12 32c.

4. What coſt $862\frac{1}{8}$ feet of boards, at D.12 per M?
Ans. D.10 34c. 6m.

5. What coſt 32159ft. boards, at D.13·75 per M.
Ans. D.442 18c. $6\frac{1}{4}$m.

BILL OF PARCELS.

Newburyport, Jan. 1ſt, 1809.

Mr. Timothy Huckſter

Bought of *Samuel Merchant*,

$25\frac{1}{2}$lb. Bohea Tea, at 3s. 6d. per lb.
48 lb. Cheeſe, at 9d. per lb.
15 Pair Worſted Hoſe, at 5s. 8d. per pair,
$4\frac{1}{2}$ Dozen Women's Gloves, at 36s. 6d. per dozen,
19 Dozen Knives and Forks, at 5s. 9d. per dozen,
9 Grindſtones, at 15s. 9d. per Stone,
$\frac{1}{4}$ Cwt. Brown Sugar, at 51s. per cwt.
31 lb. Loaf Sugar, at 1s. $0\frac{1}{2}$d. per lb.

£34 3 $3\frac{1}{2}$

Received payment in full,

Samuel Merchant.

TARE AND TRET.

TARE and Tret are practical Rules for deducting certain allowances, which are made by merchants and tradeſmen in ſelling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at ſo much per box, &c. at ſo much per cwt. or at ſo much in the groſs weight.

Tret is an allowance of 4lb. in every 104lb. for waſte, duſt, &c.

Cloff is an allowance of 2lb. upon every 3cwt.

Groſs weight is the weight of any ſort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is when part of the allowance is deducted from the groſs.

Neat weight is what remains after all allowances are made.

CASE I.*

When the tare is at ſo much per box, barrel, or bag, &c. : Multiply the number of boxes, barrels, &c. by the tare, and ſubtract the product from the groſs, and the remainder will be the neat weight required.

Examples.

1. In 6 hogſheads of ſugar, each weighing 9cwt. 2qrs. 10lb. groſs, tare 25lb. per hogſhead; how much neat?

	Cwt.	qr.	lb.	Cwt.	qr.	lb.	
25×6=	1	1	10	9	2	10	groſs weight of 1 hhd.
						6	
				57	2	4	groſs.
				1	1	10	tare.
			Ans.	56	0	22	neat.

2. In 5 bags of cotton, marked with the groſs weight as follows, tare 23lb. per bag, what neat weight?

	Cwt.	qr.	lb.
A =	7	1	19
B =	3	3	27
C =	5	1	12
D =	6	0	15
E =	8	1	0

Ans. 30cwt. 0qr. 14lb. neat.

3. What is the neat weight of 15 hogſheads of tobacco, each weighing 7cwt. 1qr. 13lb. tare 100lb per hhd.?

Ans. 97cwt. 0qr. 11lb.

CASE II.

When the tare is at ſo much per cwt. : Divide the groſs weight by the aliquot parts of a cwt. ſubtract the quotient from the groſs, and the remainder will be the neat weight.

Examples.

1. In 129cwt. 3qrs. 16lb. groſs, tare 14lb. per cwt. what neat weight?

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* This, as well as every other caſe in this rule, is only an application of the rules of Proportion and Practice.

		Cwt.	qr.	lb.	
14lb.	$\frac{1}{8}$	129	3	16	groſs.
		16	0	26½	tare.
	Ans.	113	2	17½	neat.

2. In 97cwt. 1qr. 7lb. groſs, tare 20lb. per cwt. what neat weight?

		Cwt.	qr.	lb.	
16	$\frac{1}{7}$	97	1	7	groſs.
4	$\frac{1}{4}$	13	3	17	} Add.
		3	1	25	
	Subtract	17	1	14	tare.
	Ans.	79	3	21	neat.

3. What is the neat weight of 9 barrels of pot-aſh, each weighing 305lb. groſs, tare 12lb. per cwt.?

Ans. 2450lb. 14oz. 4$\frac{4}{7}$dr.

4. What is the value of the neat weight of 7 hogſheads of tobacco, at 5l. 7s. 6d. per cwt. each weighing 8cwt. 3qrs. 10lb. groſs, tare 21lb. per cwt.?

Ans. 270l. 4s. 4½d. reckoning the odd ounces.

CASE III.

When tret is allowed with tare: Divide the ſuttle weight by 26, and the quotient will be the tret, which ſubtract from the ſuttle, and the remainder will be the neat.

EXAMPLES.

1. In 247cwt. 2qrs. 15lb. groſs, tare 28lb. per cwt. and tret 4lb. for every 104lb. what neat weight?

		Cwt.	qr.	lb.	oz.	
28	$\frac{1}{4}$	247	2	15		groſs.
		61	3	17	12	tare ſubtract.
4	$\frac{1}{26}$	185	2	25	4	ſuttle.
		7	0	16	0	tret, ſubtract.
	Ans.	178	2	9	4	neat.

2. What is the neat weight of 4 hogſheads of tobacco, weighing as follow:

	cwt.	qr.	lb.		tare lb.
The 1ſt	5cwt.	1qr.	12lb.	groſs, tare	65lb.
2d	3	0	19		75
3d	6	3	00		49
4th	4	2	9		35

and allowing tret to each as uſual?

Ans. 17cwt. 0qr. 19lb. +

CASE IV.

When tare, tret, and cloff are allowed: Deduct the tare and tret as before, and divide the ſuttle by 168, and the quotient will be the cloff, which ſubtract from the ſuttle, and the remainder will be the neat.

EXAMPLES.

1. What is the neat weight of 1hhd. of tobacco, weighing 16cwt. 2qrs. 20lb. groſs, tare 14lb. per cwt. tret 4lb. per 104, and cloff 2lb. per 3cwt.?

14lb. is $\frac{1}{8}$)	16	2	20	0	groſs.
	2	0	9	8	tare ſubtract.
4lb. is $\frac{1}{26}$)	14	2	10	8	
		2	6	13	tret, ſubtract.
2lb. is $\frac{1}{168}$)	14	0	3	11	ſuttle.
	0	0	9	5	cloff, ſubtract.
Ans.	13	3	22	6	neat.

2. If 9 hogſheads of tobacco contain 85cwt. 0qr. 2lb. tare 30lb. per hogſhead, tret and cloff as uſual, what will the neat weight come to, at 6½d. per lb. after deducting for duties and other charges 51l. 11s. 8d.?

Ans. £187 18s. 5d.

INVOLUTION,

OR TO RAISE POWERS.

A POWER is the product ariſing from multiplying any given number into itſelf continually a certain number of times, thus:

$3\times3=9$ is the 2d power, or ſquare of 3 $=3^2$
$3\times3\times3=27$ is the 3d power or cube of 3 $=3^3$
$3\times3\times3\times3=81$ is the 4th power, or the biquadrate of 3, &c. $=3^4$

The number denoting the power is called the *index*, or the *exponent* of that power. Thus, the 4th power of 3 is 81, or 3^4; the ſecond power of 5 is 25, or 5^2, &c.

$2\times2=4$, the ſquare of 2; $4\times4=16=$ 4th power of 2; $16\times16=256=$ 8th power of 2, &c.

Rule.

Multiply the given number, root, or firſt power, continually by itſelf, till the number of multiplications be 1 leſs than the index of the power to be found, and the laſt product will be the power required.

Note. Whence, becauſe fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raiſing each of their terms to the power required, and if a mixed number be propoſed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

Examples.

1. What is the 5th power of 9?

$$\begin{array}{rl} 9 & \\ 9 & \\ \hline 81 & = \text{2d power.} \\ 9 & \\ \hline 729 & = \text{3d power.} \\ 9 & \\ \hline 6561 & = \text{4th power.} \\ 9 & \\ \hline 59049 & = \text{5th power, or anſwer} = 9^5. \end{array}$$

2. What is the 5th power of $\frac{3}{5}$? Ans. $\frac{243}{3125}$.

3. What is the 4th power of ·045?
Ans. ·000004100625.

Here we ſee, that in raiſing a fraction to a higher power, we decreaſe its value.

A TABLE OF POWERS.

Roots	1	1	2	3	4	5	6	7	8	9
Squares	2	1	4	9	16	25	36	49	64	81
Cubes	3	1	8	27	64	125	216	343	512	729
Biquadrates	4	1	16	81	256	625	1296	2401	4096	6561
Surſolids	5	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes	6	1	64	729	4096	15625	46656	117649	262144	531441
Second Surſolids	7	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates ſquared	8	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes cubed	9	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surſolids ſquared	10	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Surſolids	11	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Squa. Cube ſquared	12	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

THE Root is a number, whoſe continual multiplication into itſelf produces the power, and is denominated the ſquare, cube, biquadrate, or 2d, 3d, 4th, &c. root, accordingly as it is, when raiſed to the 2d, 3d, &c. power, equal to that power. Thus, 4 is the ſquare root of 16, becauſe $4\times4=16$, and 3 is the cube root of 27, becauſe $3\times3\times3=27$, and ſo on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which preciſe roots can never be determined. But by the help of decimals, we can approximate towards the root to any aſſigned degree of exactneſs.

The roots, which aproximate, are called *ſurd roots*; and thoſe which are perfectly accurate, are called *rational roots*.

Roots are ſometimes denoted by writing the character $\sqrt{}$ before the power, with the index of the power over it; thus the 3d root of 36 is expreſſed $\sqrt[3]{36}$, and the 2d root of 36 is $\sqrt{36}$, the index 2 being omitted when the ſquare root is deſigned.

If the power be expreſſed by ſeveral numbers, with the ſign + or — between them, a line is drawn from the top of the ſign over all the parts of it. Thus the 3d root of 47+22 is $\sqrt[3]{47+22}$, and the 2d root of 59—17 is $\sqrt{59-17}$, &c.

Sometimes roots are deſigned like powers, with fractional indices. Thus the ſquare root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and the 4th root of 37—20 is $\overline{37-20}^{\frac{1}{4}}$, &c.

EXTRACTION OF THE SQUARE ROOT.

Rule.

1. Diſtinguiſh the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and ſo on, which points ſhew the number of figures the root will conſiſt of.

2. Find the greateſt ſquare number in the firſt, or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in diviſion) for the firſt figure of the root, and the ſquare number under the period, and ſubtract it therefrom, and to the remainder bring down the next period for a dividend.

3 Place the double of the root, already found, on the left hand of the dividend for a diviſor.

4. Seek how often the diviſor is contained in the dividend, (except the right hand figure) and place the anſwer in the root for the ſecond figure of it, and likewiſe on the right hand of the diviſor: Multiply the diviſor, with the figure laſt annexed, by the figure laſt placed in the root, and ſubtract the product from the dividend: To the the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new diviſor, (or bring down your laſt diviſor for a new one, doubling the right hand figure of it) and from theſe find the next figure in the root as laſt directed; and continue the operation in the ſame manner till you have brought down all the periods.

Note 1. If when the given power is pointed off as the power requires, the left hand period ſhould be deficient, it muſt nevertheleſs ſtand as the firſt period.

Note 2. If there be decimals in the given number, it muſt be pointed both ways from the place of units: If when there are integers, the firſt period in the decimals be deficient it may be completed by annexing ſo many cyphers as the power requires: And the root muſt be made to conſiſt of ſo many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhauſted, the operation may be continued at pleaſure by annexing cyphers.

Examples.

1. Required the ſquare root of 30138696025.

30138696025(173605 the root.
1

1ſt Diviſor=27)201
189

2d Diviſor=343)1238
1029

3d Diviſor=3466)20969
20796

4th Diviſor=347205)1736025
1736025

2. Required the ſquare root of 575·5.

575·50(23·98 +, root.
4

43)175
129

469)4650
4221

4788)42900
38304

4596 Remainder.

3. What is the ſquare root of 10342656?
Ans. 3216.

4. What is the ſquare root of 964·5192360241?
Ans. 31·05671.

5. What is the ſquare root of ·0000316969?
Ans. ·00563.

Note. When more than half the root is found, the remaining figures of it may be found by Diviſion, making uſe of the laſt diviſor, and bringing down ſo many of the next figures of the reſolvend, as there are periods to come down when you began the diviſion.

Rules *for the Square Root of Vulgar Fractions and Mixed Numbers.*

After reducing the fraction to its loweſt terms, for this and all other roots; then,

1. Extract the *root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the beſt method, provided the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required: —Or.

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers may either be reduced to improper fractions and extracted by the firſt or ſecond rule; or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Examples.

1. What is the ſquare root of $\frac{144}{15129}$?

By Rule 1.

$\frac{144}{15129}=\frac{16}{1681}$ $\quad$ 16(4 root of the numerator.
16

1681(41 root of the denominator.
16

81)81 Therefore, $\frac{4}{41}$ = the root of the given fraction.
81

By Rule 2.

$16\times1681=26896$, and $\sqrt{26896}=164$. Then

$\frac{164}{1681}=\frac{16}{164}=\frac{4}{41}=\cdot09756+$

By Rule 3.

1681)16(·0095181439+ And $\sqrt{·0095181439}$=·09756+.

2. What is the ſquare root of $\frac{2793}{8208}$? Ans $\frac{7}{12}$.

3. What is the ſquare root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.

Note. In extracting the ſquare or cube root of any ſurd number there is always a remainder or fraction left when the root is found: To find the value of which, the common method is to annex pairs of cyphers to the reſolvend for the ſquare, and ternaries of cyphers to that of the cube, which makes it tedious to diſcover the value of the remainder, eſpecially in the cube. Now this trouble may be ſaved by the following method.

In the ſquare the quotient is always doubled for a new diviſor: Therefore when the work is completed, the root doubled is the true diviſor, or denominator* to its own fraction; as, if the root be 12, the denominator will will be 24; to be placed under the remainder; which vulgar fraction, or its equivalent decimal, muſt be annexed to the quotient, or root, to complete it.

If to the remainder, either of the ſquare or cube, cyphers be annexed, and divided by their reſpective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION AND USE OF THE SQUARE ROOT.

Problem I. *To find a mean proportional between two numbers.*

Rule. Multiply the given numbers together, and extract the ſquare root of the product; which root will be the mean proportional ſought.

Example.

What is the mean proportional between 24 and 96?

$\sqrt{96 \times 24}$=48 Anſwer.

---

[*] Theſe denominators give a ſmall matter too much in the ſquare root, and too little in the cube, yet they will be ſufficient in common uſe.

Prob. II. *To find the ſide of a* ſquare *equal in* area *to any given* ſuperficies *whatever.*

Rule. Find the *area*, and the ſquare root is the ſide of the ſquare ſought.

Examples.

1. If the area of a circle be 184·125, what is the ſide of a ſquare equal in area thereto?

$\sqrt{184{\cdot}125}=13{\cdot}569+$ Anſwer.

2. If the area of a triangle be 160, what is the ſide of a ſquare equal in area thereto?

$\sqrt{160}=12{\cdot}649+$ Anſwer.

Prob. III. A certain general has an army of 5625 men: Pray, how many muſt he place in rank and file, to form them into a ſquare? $\sqrt{5625}=75$ Ans.*

Prob. IV. Let 1652 men be ſo formed, as that the number in rank may be double the file.

$\sqrt{\frac{10952}{2}}=74$ in file, and $74\times2=148$ in rank.

Prob. V. If it be required to place 2016 men ſo as that there may be 56 in rank and 36 in file, and to ſtand 4 feet diſtance in rank, and as much in file; how much ground do they ſtand on?

To anſwer this, or any of the kind, uſe the following proportion:—As unity : to the diſtance :: ſo is the number in rank leſs by one : to a fourth number;—next do the ſame by the file, and multiply the two numbers together, found by the above proportion, and the product will be the anſwer.†

As 1 : 4 :: 56—1 : 220. And, as 1 : 4 :: 36—1 : 140. Then, $220\times140=30800$ ſquare feet, the anſwer.

Prob. VI. Suppoſe I would ſet out an orchard of 600 trees, ſo that the length ſhall be to the breadth, as 3 to 2, and the diſtance of each tree, one from the other,

---

* If you would have the number of men double, triple, or quadruple, &c. as many in rank as in file, extract the ſquare root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, triple qadruple, &c. and the product will be the number in rank.

† The above rule will be found uſeful in planting trees, having the diſtance of ground between each given.

7 yards ; how many trees muſt it be in length, and how many in breadth ; and how many ſquare yards of ground do they ſtand on ?

To reſolve any queſtion of this nature, ſay, as the ratio in length : is to the ratio in breadth :: ſo is the number of trees : to a fourth number ; whoſe ſquare root is the number in breadth ; And as the ratio in breadth : is to the ratio in length :: ſo is the number of trees to a fourth, whoſe root is the number in length.

As 3 : 2 :: 600 : 400. And $\sqrt{400}=20=$number in breadth.

As 2 : 3 :: 600 : 900. And $\sqrt{900}=30=$number in length.

As 1 : 7 :: 30−1 : 203. And as 1 : 7 :: 20−1 : 133. And 203×133=26999 ſquare yards, Ans.

Prob VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a ciſtern in 3 hours ; I demand the diameter of another pipe, which will fill the ſame ciſtern in 1 hour.

Rule. As the given time is to the ſquare of the given diameter, ſo is the required time to the ſquare of the required diameter.

$\frac{3}{4}=\cdot75$: and $\cdot75\times\cdot75=\cdot5625$. Then, as 3h. : ·5625 :: 1h. : 1·6875 inverſely, and $\sqrt{1\cdot6875}=1\cdot3$ inch nearly, Anſwer.

Prob VIII If a pipe, whoſe diameter is 1·5 inch, fill a ciſtern in 5 hours, in what time will a pipe, whoſe diameter is 3·5 inches, fill the ſame ?

$1\cdot5\times1\cdot5=2\cdot25$; and $3\cdot5\times3\cdot5=12\cdot25$. Then, as 2·25 : 5 :: 12·25 : ·918+ hour inverſely,=55 minutes 5 ſeconds. Anſwer.

Prob. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water ; in what time will 3 pipes, each 4 inches bore, be in diſcharging double the quantity ?

$6\times6=36$ $4\times4=16$, and $16\times3=48$. Then, as 36 : 4h. :: 48 : 3h. inverſely, and as 1w. : 3h. :: 2w. : 6h. Anſwer.

Prob. X. Given the diameter of a circle to make another circle, which ſhall be 2, 3, 4, &c. times greater or leſs than the given circle.

Rule.—Square the given diameter, and if the required circle be greater, multiply the ſquare of the diameter by the given proportion, and the root of the product will be the required diameter ;—But if the required circle be leſs ; divide the ſquare of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle, whoſe diameter is 4 inches : I demand the diameter of a circle 3 times as large.

$4\times4=16$; and $16\times3=48$; and $\sqrt{48}=6{\cdot}928+$ inches, Anſwer.

Prob. XI. To find the diameter of a circle equal in area to an ellipſis (or oval) whoſe tranſverſe and conjugate diameters are given.*

Rule.—Multiply the two diameters of the ellipſis together ; and the Square Root of that product will be the diameter of a circle equal to the ellipſis.

Let the tranſverſe diameter of an ellipſis be 48, and the conjugate 36 : What is the diameter of an equal circle ?

$48\times36=1728$, and $\sqrt{1728}=41{\cdot}569+$ the Anſwer.

Note. The ſquare of the hypothenuſe, or the longeſt ſide of a right angled triangle, (by 47th B. 1. Euc.) is equal to the ſum of the ſquares of the other two ſides ; and conſequently the difference of the ſquares of the hypothenuſe and either of the other ſides is the ſquare of the remaining ſide.

Prob. XII. A line 36 yards long will exactly reach from the top of a fort to the oppoſite bank of a river, known to be 24 yards broad. The height of the wall is required ?

$36\times36=1296$; and $24\times24=576$. Then, $1296-576=720$, and $\sqrt{720}=26{\cdot}83+$ yards, the Anſwer.

Prob. XIII. The height of a tree, growing in the centre of a circular iſland 44 feet in diameter, is 75 feet, and a line ſtretched from the top of it over to the hither edge of the water is 256 feet. What is the breadth of the ſtream, provided the land on each ſide of the water be level ?

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* The tranſverſe and conjugate are the longeſt and ſhorteſt diameters of an ellipſis ; they paſs through the centre, and croſs each other at right angles.

$256\times256=65536$; and $75\times75=5625$: Then $65536-5625=59911$, and $\sqrt{59911}=244\cdot76+$ and $244\cdot76-\frac{44}{2}=222\cdot76$ feet, Anſwer.

PROB. XIV. Suppoſe a ladder 60 feet long be ſo planted as to reach a window 37 feet from the ground, on one ſide of the ſtreet, and without moving it at the foot, will reach a window 23 feet high on the other ſide; I demand the breadth of the ſtreet.

$60\times60=3600$. $37\times37=1369$. $23\times23=529$. Then, $3600-1369=2231$, and $\sqrt{2231}=47\cdot23+$, and $3600-529=3071$, and $\sqrt{3071}=55\cdot41+$, then $47\cdot23+55\cdot41=102\cdot64$ feet, Ans.

PROB. XV. Two ſhips ſail from the ſame port; one goes due north 45 leagues, and the other due weſt 76 leagues: How far are they aſunder?

$45\times45=2025$. $76\times76=5776$. Then, $5776+2025=7801$ and $\sqrt{7801}=88\cdot32$ leagues, the Anſwer.

EXTRACTION OF THE CUBE ROOT.

A Cube is any number multiplied by its ſquare. To extract the cube root, is to find a number which being multiplied into its ſquare, ſhall produce the given number

RULE.

1. Separate the given number into periods of three figures each, by putting a point over the unit figure and every third figure beyond the place of units.

2. Find the greateſt cube in the left hand period, and put its root in the quotient.

3. Subtract the cube thus found, from the ſaid period, and to the remainder bring down the next period, and call this the dividend.

4. Multiply the ſquare of the quotient by 300, calling it the triple ſquare, and the quotient by 30 calling it the triple quotient, and the ſum of theſe call the diviſor.

5. Seek how often the diviſor may be had in the dividend, and place the reſult in the quotient.

6. Multiply the triple ſquare by the laſt quotient figure, and write the product under the dividend; multiply the ſquare of the laſt quotient figure by the triple quotient,

and place this product under the laſt ; under all, ſet the cube of the laſt quotient figure and call their ſum the ſubtrahend.

7. Subtract the ſubtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and ſo on till the whole be finiſhed.

Note. The ſame rule muſt be obſerved for continuing the operation and pointing for decimals, as in the ſquare root.

Examples.

1. Required the cube root of 436036824287.

436036824287(7583 the root.
343

1ſt Diviſor=14910)93036=1ſt Dividend.

73500
5250
125

78875=1ſt Subtrahend.

2d Diviſor=1689750)14161824=2d Dividend.

13500000
144000
512

13644512=d Subtrahend.

3d Diviſor=172391940)517312287=3d Dividend.

517107600
204660
27

517312287=3d Subtrahend.

. . . .

The Method of Operation.

7×7×300 = 14700 = 1ſt Triple ſquare.
7×30 = 210 = 1ſt Triple quotient.

14910 = 1ſt Diviſor.

14700×5 = 73500
5×5×210 = 5250
5×5×5 = 125

78875 = 1ſt Subtrahend.

75×75×300 = 1687500 = 2d Triple ſquare.
75×30 = 2250 = 2d Triple quotient.

1689750 = 2d Diviſor.

1687500×8 = 13500000
2250×8×8 = 144000
8×8×8 = 512

13644512 = 2d Subtrahend.

758×758×300 = 172369200 = 3d Triple ſquare.
758×30 = 22740 = 3d Triple quotient.

172391940 = 3d Diviſor.

172369200×3 = 517107600
22740×3×3 = 204660
3×3×3 = 27

517312287 = 3d Subtrahend.

2. What is the cube root of 34965783? Ans. 327.
3. What is the cube root of 84·604519? Ans. 4·39.
4. What is the cube root of ·008649? Ans. ·2052+.
5. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finiſhed.

In the extraction of the cube root, the quotient is ſaid to be ſquared and tripled for a new diviſor: But is not really ſo, till the triple number of the quotient be added to it; therefore, when the operation is finiſhed, it is but ſquaring the quotient, or root, then multiplying it by 3, and to that number adding the triple number of the root, when it will become the diviſor, or true denominator to its own fraction, which fraction muſt be annexed to the quotient to complete the root.

Suppoſe the root to be 12, when ſquared it will be 144, and multiplied by 3 it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.

BY APPROXIMATION.

1. Aſſume a root, whoſe cube will probably be near the given number; then multiply the ſquare of the aſſumed root by 3, and divide the reſolvend by this product; to this quotient add $\frac{2}{3}$ of the aſſumed root, and the ſum will be the true root, or an approximation to it.

2. For each ſucceeding operation let the laſt approximated root be the aſſumed root, and proceeding in this manner, the root may be extracted to any aſſigned exactneſs.

1. What is the cube root of 7?

Let the aſſumed root be 2. Then $\overline{2\times2\times3}=12$ the diviſor.

12)7·0(·583; to this add $\frac{2}{3}$ of 2=1·333, &c. that is, ·583+1·333=1·916 approximated root.

Now aſſume 1·916 for the root. Then, by the ſecond proceſs, the root is $\frac{7}{3\times\overline{1\cdot916}^2}+\frac{2}{3}\times\overline{1\cdot916}=1\cdot9126$, &c.

2. What is the cube root of 9? Let 2 be the aſſumed root as before. Then, $\frac{9}{12}+\frac{2}{3}\times2=2\cdot08$ the approximated root. Now aſſume 2·08. Then $\frac{9}{3\times\overline{2\cdot08}^2}+\overline{\frac{2}{3}\times2\cdot08}$ $=2.08008$, &c.

3. What is the cube root of 282? Let 6 be the aſſumed root. Then 6×6×3=108)282(2·611, &c. and 2·611+$\frac{2}{3}$ of 6=6·611 approximated root. Now aſſume 6·611, and it will be 6·611×6·611×3=131·116)282(2·1507, &c. and 2·1507+$\frac{2}{3}$ of 6·611=6·558 a farther approximated root.

4. What is the cube root of 1728? Here the aſſumed root is 10. Then, 10×10×3=300)1728(5·76, and 5·76+$\frac{2}{3}$ of 10=12·426. Now aſſume 12·426, then 12·426×12·426×3=426·216428)1728(3·732, and 3·732+$\frac{2}{3}$ of 12·426=12·014 a farther approximated root, and ſo on.

APPLICATION AND USE OF THE CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

Rule.

1. Divide the *greater* by the *leſs*, and extract the cube root of the quotient.

2. Multiply the root, ſo found, by the leaſt of the given numbers, and the product will be the *leaſt*.

3. Multiply this product by the ſame root, and it will give the greateſt.

Examples.

1. What are the two mean proportionals between 6 and 750?

750÷6=125, and $\sqrt[3]{125}=5$. Then 5×6=30=leaſt, and 30×5=150=greateſt. Ans. 30 and 150.
Proof. As 6 : 30 :: 150 : 750.

2. What are the two mean proportionals between 56 and 12096? Ans. 336 and 2016.

Note. The ſolid contents of ſimilar figures are in proportion to each other, as the cubes of their ſimilar ſides or diameters.

3. If a bullet 6 inches diameter weigh 32lb.; what will a bullet of the ſame metal weigh whoſe diameter is 3 inches?

6×6×6=216. 3×3×3=27.

As 216 : 32lb. :: 27 : 4lb. Ans.

4. If a globe of ſilver of 3 inches diameter be worth £45, what is the value of another globe, of a foot diameter?   3×3×3=27.   12×12×12=1728.

As 27 : 45 :: 1728 : £2880 Ans.

2d. The ſide of a cube being given, to find the ſide of that cube which ſhall be double, triple, &c. in quantity to the given cube.

RULE. Cube your given ſide, and multiply it by the given proportion between the given and required cube, and the cube root of the product will be the ſide ſought.

5. If a cube of ſilver, whoſe ſide is 4 inches, be worth £50, I demand the ſide of a cube of the like ſilver, whoſe value ſhall be 4 times as much?

4×4×4=64, and 64×4=256 $\sqrt[3]{256}$=6·349+inches, Ans.

6. There is a cubical veſſel, whoſe ſide is 2 feet; I demand the ſide of a veſſel, which ſhall contain three times as much?

2×2×2=8, and 8×3=24. $\sqrt[3]{24}$=2·884=2ft. $10\frac{3}{8}$in. Ans.

7.* The diameter of a buſhel meaſure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the ſide of a cubick box, which ſhall contain that quantity?

Ans 12·907+ inches.

8. Suppoſe a ſhip of 500 tons has 89 feet keel, 36 feet beam, and 16 feet deep in the hold; what are the dimenſions of a ſhip of 200 tons, of the ſame mould and and ſhape?

89×89×89=704969=cubed keel.

As 500 : 200 :: 704969 : 281987·6 cube of the required keel.

$\sqrt[3]{281987{\cdot}6}$=65·57 feet the required keel.

As 89 : 65·57 :: 36 : 26·528=$26\frac{1}{2}$ feet beam. nearly.

As 89 : 65·57 :: 16 : 11·7 feet, depth of the hold, nearly.

3d. From the proof of any cable to find the ſtrength of any other.

RULE. The ſtrength of cables, and conſequently the weights of their anchors, are as the cubes of their peripheries.

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* Multiply the ſquare of the diameter by 7854, and the product by the height; the cube root of the laſt product is the anſwer.

9. If a cable 12 inches about require an anchor of 18 cwt. of what weight muſt an anchor be for a 15 inch cable?

Cwt. Cwt.

As 12×12×12 : 18 :: 15×15×15 : 35·15625, Ans.

10. If a 15 inch cable require an anchor 35·15625cwt. what muſt the circumference of a cable be, for an anchor of 18cwt.?

As 35·15625 : 15×15×15 :: 18 : 1728, and $\sqrt[3]{1728}$ =12, Anſwer.

EXTRACTION OF THE BIQUADRATE ROOT.

RULE.

Extract the ſquare root of the reſolvend, and then the ſquare root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

```
  .  .  .            . .
  20736(144          144(12 root required.
  1                  1
  ----               --
24)107            22)44
   96                44
   ----              --
284)1136
    1136
```

*A GENERAL RULE**

FOR EXTRACTING THE ROOTS OF ALL POWERS.

1. Prepare the given number for extraction, by pointing off from the unit's place, as the required root directs.

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* The extracting of roots of very high powers will, by this rule, be a tedious operation:—The following method, when practicable, will be much more convenient.

When the index of the power, whoſe root is to be extracted, is a compoſite number, take any two or more indices, whoſe product is equal to the given index, and extract out of the given number a root anſwering to another of the indices, and ſo on to the laſt.

Thus, the fourth root=ſquare root of the ſquare root; the ſixth root=ſquare root of the cube root;—the eighth root=ſquare root of the fourth root;—the ninth root=the cube root of the cube root;—the tenth root=ſquare root of the fifth root;—the twelfth root=cube root of the fourth, &c.

2. Find the firſt figure of the root by trial, or by inſpection into the Table of Powers, and ſubtract its power from the left hand period.

3. To the remainder bring down the firſt figure in the next period, and call it the *dividend.*

4. Involve the root to the next inferiour power to that which is given, and multiply it by the number, denoting the given power, for a *diviſor.*

5 Find how many times the diviſor may be had in the dividend, and the quotient will be another figure of the root

6. Involve the whole root to the given power, and ſubtract it from the *given number* as before.

7. Bring down the firſt figure of the next period to the remainder for a new dividend, to which find a new diviſor, as before, and, in like manner, proceed till the whole be finiſhed.

EXAMPLES.

1. What is the cube root of 20346417?

$\dot{2}03\dot{4}64\dot{1}7(273$

$2^3 = 8$ = 1ſt Subtrahend.

$2^2\times3=12)123$ = Dividend.

$27^3 = 19683$ = 2d Subtrahend.
$27^2\times3=2187)6634$ = 2d Dividend.

$273^3 = 20346417$ = 3d Subtrahend.

. . . .

$2\times2\times2=8$ root of the 1ſt period, or 1ſt ſubtrahend.
$2\times2=4$ (=next inferiour power) and,
4×3 (the index of the given power)=12=1ſt diviſor.
$27\times27\times27=19683$=2d ſubtrahend.
$27\times27=729$ (=next inferiour power) and,
729×3 (=index of the given power)=2187=2d div.
$273\times273\times273=27346417$=3d ſubtrahend.

2. What is the biquadrate root of 34827998976?

Ans. 431·9+.

3. Extract the ſquare cubed, or ſixth root of 1178420166015625. Ans. 325.

OF PROPORTION IN GENERAL.

NUMBERS are compared together, to diſcover the relations they have to each other.

There muſt be two numbers to form a compariſon: The number, which is compared, being written firſt, is called the *antecedent*; and that, to which it is compared, the *conſequent*.

Numbers are compared with each other two different ways: The one compariſon conſiders the *difference* of the two numbers, and is called Arithmetical Relation, the difference being ſometimes named the Arithmetical Ratio; and the other conſiders their *quotient*, which is termed Geometrical Relation, and the quotient, the Geometrical Ratio. Thus, of the numbers 12 and 4, the difference, or Arithmetical Ratio, is 12—4=8; and the Geometrical ratio is $\frac{12}{4}=3$.[*]

If two or more couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms, ſimilarly poſited, that is, either all the greater, or all the leſs taken as antecedents, and the reſt as conſequents, are called proportionals. So, the two couplets, 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.[†]

Proportion is diſtinguiſhed into continued and diſcontinued. If, of ſeveral couplets of proportionals, written down in a ſeries, the difference or ratio of each conſequent, and the antecedent of the next following couplet, be the ſame as the common difference or ratio of the couplets, the proportion is ſaid to be continued, and the numbers themſelves, a ſeries of continued arithmetical or geometrical proportionals. So 2, 4, 6, 8, form an arithmetical progreſſion; for $4-2=6-4=8-6$; and 2, 4, 8, 16, a geometrical progreſſion; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

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[*] RATIOS are, here, always conſidered as the reſult of the greater term of compariſon diminiſhed, or divided by the leſs; not regarding which of them be the antecedent.

[†] Four numbers are ſaid to be *reciprocally* or *inverſely* proportional, when the fourth is leſs than the ſecond by as many times as the third is greater than the firſt, or when the firſt is to the third, as the fourth to the ſecond, and *vice verſa*. Thus, 2, 9, 6, and 3 are reciprocal proportionals.

NOTE. It is common to read the geometricals 2 : 4 :: 8 : 16. thus, 2 is to 4 as 8 to 16; or, as 2 to 4 ſo is 8 to 16.

But, if the difference or ratio of the conſequent of one couplet, and the antecedent of the next couplet be not the ſame as the common difference or ratio of the couplets, the proportion is ſaid to be diſcontinued. So 4, 2, 8, 6, are in diſcontinued arithmetical proportion; for $4-2=8-6=2$ =common difference of the couplets, $8-2=6$=difference of the conſequent of one couplet and the antecedent of the next; alſo, 4, 2, 16, 8 are in diſcontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2=$ common ratio of the couplets, and $\frac{16}{2}=8$=ratio of the conſequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

THEOREM 1. If any four quantities, 2, 4, 6, 8, be in Arithmetical Proportion, the ſum of the two means is equal to the ſum of the two extremes.

And if any three quantities, 2, 4, 6, be in Arithmetical Proportion, the double of the mean is equal to the ſum of the extremes.

THEOREM 2. In any continued arithmetical proportion, 1, 3, 5, 7, 9, 11, the ſum of the two extremes, and and that of every other two terms, equally diſtant from them, are equal: Thus, $1+11=3+9=5+7$.

When the number of terms is odd, as in the proportion, 3, 8, 13, 18, 23, then, the ſum of the two extremes, being double to the mean, or middle term, the ſum of any other two terms equally remote from the extremes muſt likewiſe be double to the mean.

Theorem 3. In any continued arithmetical proportion, 4, 4+2, 4+4, 4+6, 4+8, &c. the laſt or greateſt term is equal to the firſt or leaſt more the common difference of the terms drawn into the number of all the terms after the firſt, or into the whole number of the terms leſs one.

Theorem 4. The ſum of any rank, or ſeries, of quantities in continued arithmetical proportion, 1, 3, 5, 7, 9, 11, is equal to the ſum of the two extremes multiplied into half the number of terms.*

ARITHMETICAL PROGRESSION.

ANY rank of numbers, more than two, increaſing by a common exceſs, or decreaſing by a common difference, is ſaid to be in Arithmetical Progreſſion.

If the ſucceeding terms of a progreſſion exceed each other, it is called an aſcending ſeries or progreſſion; if the contrary, a deſcending ſeries.

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* The ſame thing alſo holds, when the number of terms is odd, as in the ſeries, 4, 8, 12, 16, 20; for then the mean, or middle term, being equal to half the ſum of any two terms equally diſtant from it on contrary ſides, it is obvious that the value of the whole ſeries is the ſame as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the ſum of the two extremes) multiplied by the whole number of terms; or to the ſum of the extremes multiplied by half the number of terms.

The ſum of any number of terms of the arithmetical ſeries of odd numbers, 1, 3, 5, 7, 9, &c. is equal to the ſquare of that number.

For, $0+1$ or the ſum of 1 term$=1^2$ or 1
$1+3$ or the ſum of 2 terms$=2^2$ or 4
$4+5$ or the ſum of 3 terms$=3^2$ or 9
$9+7$ or the ſum of 4 terms$=4^2$ or 16
$16+9$ or the ſum of 5 terms$=5^2$ or 25, &c.

Example.

The firſt term, the ratio, and number of terms given, to find the ſum of the ſeries.

A Gentleman travelled 29 days; the firſt day he went but 1 mile, and increaſed every day's travel 2 miles: How far did he travel?

$29\times29=841$ miles the anſwer.

So { 0, 2, 4, 6, 8, 10, &c. is an aſcending arithmetical ſeries.
1, 2, 4, 8, 16, 32, &c. is an aſcending geometrical ſeries. }

And { 10, 8, 6, 4, 2, 0, &c. is a deſcending arithmetical ſeries.
32, 16, 8, 4, 2, 1, &c. is a deſcending geometrical ſeries. }

The numbers, which form the ſeries, are called the *terms* of the progreſſion.

NOTE. The firſt and laſt terms of a progreſſion are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be eaſily found.

1. The firſt term.
2. The laſt term.
3. The number of terms.
4. The common difference.
5. The ſum of all the terms.

PROBLEM I.

The firſt term, the laſt term and the number of terms being given, to find the common difference.

RULE.

Divide the difference of the extremes by the number of terms leſs 1 and the quotient will be the common difference ſought.

EXAMPLES.

1. The extremes are 3 and 39, and the number of terms is 19: What is the common difference?

$$\left.\begin{array}{r} 39 \\ -\ 3 \end{array}\right\} \text{Ext.}^s$$

Divide by the number of terms—1=19—1=18)36(2 Ans.
18

Or, $\dfrac{39-3}{19-1}=2$

2. A man had 10 ſons, whoſe ſeveral ages differed alike; the youngeſt was 3 years old, and the eldeſt 48; What was the common difference of their ages?

$$\frac{48-3}{10-1}=5 \text{ Ans.}$$

3. A man is to travel from Boſton to a certain place in 9 days, and to go but 5 miles the firſt day, increaſing each day by an equal exceſs, ſo that the laſt day's journey may be 37 miles; required the daily increaſe.

$$\frac{37-5}{9-1}=4 \text{ Ans.}$$

PROBLEM II.

The firſt term, the laſt term, and the number of terms being given, to find the ſum of all the terms.

RULE. Multiply the ſum of the extremes by the number of terms, and half the product will be the anſwer.

EXAMPLES.

1. The extremes of an arithmetical ſeries are 3 and 39, and the number of terms 19; required the ſum of the ſeries?

$$\left.\begin{array}{r}39\\+\ 3\end{array}\right\}\text{Extremes.}$$

$$\begin{array}{rr}
\text{Sum} = & 42\\
\text{Number of terms} = & \times 19\\
\hline
 & 378\\
 & 42\\
\hline
 & 2)798\\
\hline
 & 399 \text{ Ans.}
\end{array}$$

Or, $\frac{\overline{39+3}\times 19}{2}=399$

2. It is required to find how many ſtrokes the hammer of a clock would ſtrike in a week, or 168 hours, provided it increaſed 1 at each hour?

$$\frac{\overline{168+1}\times 168}{2}=14196 \text{ Anſwer.}$$

3. Suppoſe a number of ſtones were laid a yard diſtant from each other for the ſpace of a mile, and the firſt a yard from a baſket; what length of ground will the

man travel over who gathers them up ſingly, returning with them one by one to the baſket?

$$\frac{\overline{3526+2}\times1760}{2}=3099360 \text{ yards}=1761 \text{ miles, Ans.}$$

N B. In this queſtion, there being 1760 yards in a mile, and the man returning with each ſtone to the baſket, his travel will be doubled; therefore the firſt term will be 2 and the laſt 1760×2; and the number of terms 1760.

4. A man bought 25 yards of linen in arithmetical progreſſion; for the 4th yard he gave 12 cents, and for the laſt yard 75 cents. What did the whole amount to, and what did it average per yard?

$\frac{75-12}{22-1}=3$ the common difference, by which the firſt term is found to be 3.

Then $\frac{\overline{75+3}\times25}{2}=$ 9D. 75c. and the average price is 39 cents per yard.

5. Required the ſum of the firſt 1000 numbers in their natural order?

$$\frac{\overline{1000+1}\times1000}{2}=500500 \text{ Ans.}$$

PROBLEM III.

Given the extremes and the common difference, to find the number of terms.

RULE. Divide the difference of the extremes by the common difference, and the quotient increaſed by 1 will be the number of terms required.

EXAMPLES.

1. The extremes are 3 and 39, and the common difference 2: What is the number of terms?

$$\left.\begin{array}{r}39\\-3\end{array}\right\}\text{Extremes.}$$

Common difference=2)36

Quotient=18

Add 1

19 Anſwer.

$$\text{Or, } \frac{39-3}{2}+1=19.$$

2. A man going a journey travelled the firſt day 7 miles, the laſt day 51 miles, and each day increaſed his journey by 4 miles: How many days did he travel, and how far?

$$\frac{51-7}{4}+1=12 \text{ days, and } \frac{\overline{51+7}\times 12}{2}=348 \text{ miles, Ans.}$$

PROBLEM IV.

The extremes and common difference given, to find the ſum of the ſeries.

RULE. Multiply the ſum of the extremes by their difference increaſed by the common difference, and the product divided by twice the common difference will give the ſum.

EXAMPLES.

1. If the extremes are 3 and 39, and the common difference 2, what is the ſum of the ſeries?

39+3=42=ſum of the extremes.

39—3=36=difference of the extremes.

36+2=38=difference of the extremes increaſed by the common difference.

$$\begin{array}{r} 42 \\ \times 38 \\ \hline 336 \\ 126 \\ \hline \text{Twice the common difference}=4)\,1596 \\ \hline 399 \text{ Ans.} \end{array}$$

$$\text{Or, } \frac{\overline{39+3}\times\overline{39-3+2}}{2\times 2}=399.$$

2. A owes B a certain ſum, to be diſcharged in a year, by paying 6d. the firſt week, 18d. the ſecond, and thus to increaſe every weekly payment by a ſhilling, till the laſt payment be £2 11s. 6d. what is the debt?

$$\frac{\overline{51\cdot5+\cdot5}\times\overline{51\cdot5-\cdot5+1}}{1\times 2}=£67\ 12s. \text{ Ans.}$$

THEOREM 5. If four quantities, 2, 6, 4, 12, are directly proportional,

Then,
1. Directly, $2 : 6 :: 4 : 12$
2. Inverſely, $6 : 2 :: 12 : 4$
3. Alternately, $2 : 4 :: 6 : 12$
4. Compoundedly, $2 : 6+2 :: 4 : 12+4$
5. Dividedly, $2 : 6-2 :: 4 : 12-4$
6 Mixtly, $6+2 : 6-2 :: 12+4 : 12-4$
7. By Multiplication, $2r : 6r :: 4 : 12$
or $2\times5 : 6\times5 :: 4 : 12$
8. By Diviſion, $\frac{2}{r} : \frac{6}{r} :: 4 : 12$
or, $\frac{2}{5} : \frac{6}{5} :: 4 : 12$

Becauſe the product of the means in each caſe, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6. If three numbers, 2, 4, 8, be in continued proportion, the ſquare of the firſt will be to *that* of the ſecond, as the firſt number to the third; that is, $2\times2 : 4\times4 :: 2 : 8$.

THEOREM 7. In any continued geometrical proportion, (1, 3, 9, 27, 81, &c) the product of the two extremes, and *that* of every other two terms, equally diſtant from them, are equal.

THEOREM 8. The ſum of any number of quantities, in continued geometrical proportion, is equal to the difference of the rectangle of the ſecond and laſt terms, and the ſquare of the firſt divided by the difference of the firſt and ſecond terms.

GEOMETRICAL PROGRESSION.

A GEOMETRICAL Progreſſion is when a *Rank*, or *Series*, of numbers, increaſes or decreaſes by the continual multiplication, or diviſion, of ſome equal number.

PROBLEM I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical ſeries, to find the other extreme.

RULE. Multiply, or divide (as the caſe may require) the given extreme by ſuch power of the ratio, whoſe exponent* is equal to the *number* of terms leſs 1, and the product, or quotient, will be the other extreme.

EXAMPLES.

1. If the firſt term be 4, the ratio 4, and the number of terms 9, what is the laſt term?

---

* As the *laſt term*, or any term near the laſt, is very tedious to be found, by continual multiplication, it will often be neceſſary, in order to aſcertain them, to have a ſeries of numbers in arithmetical proportion, called *indices* or *exponents*, beginning with a cypher, or an unit, whoſe common difference is 1.

When the *firſt* term of the ſeries and the *ratio* are *equal*, the *indices* muſt begin with an unit, and, in this caſe, the product of any two terms is equal to that term, ſignified by the *ſum* of their *indices*.

$$\text{Thus,}\left\{\begin{array}{l}1,\ 2,\ 3,\ \ 4,\ \ 5,\ \ 6,\ \text{\&c. indices or arithmetical ſeries.}\\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ \text{\&c. geometrical ſeries (leading terms)}\end{array}\right.$$

Now 6+6=12=the index of the 12th term, and
64×64=4096=the twelfth term.

But, when the *firſt* term of the ſeries and the *ratio* are *different*, the *indices* muſt begin with a cypher, and the ſum of the *indices*, made choice of, muſt be *one leſs* than the *number of terms*, given in the queſtion; becauſe 1 in the *indices* ſtands over the *ſecond term*, and 2 in the *indices*, over the *third* term, &c. And, in this caſe, the *product* of any *two* terms, divided by the *firſt*, is equal to that term *beyond* the firſt, ſignified by the *ſum* of their *indices*.

$$\text{Thus,}\left\{\begin{array}{l}0,\ 1,\ 2,\ \ 3,\ \ 4,\ \ \ 5,\ \ \ 6,\ \text{\&c. indices.}\\ 1,\ 3,\ 9,\ 27,\ 81,\ 243,\ 729,\ \text{\&c. geometrical ſeries.}\end{array}\right.$$

Here, 6 + 5 = 11 the index of the 12th term
729×243=177147 the 12th term, becauſe the firſt term of the ſeries and the ratio are different, by which mean a cypher ſtands over the firſt term.

Thus, by the help of theſe indices, and a few of the firſt terms in any geometrical ſeries, any term, whoſe diſtance from the firſt term is aſſigned, though it were ever ſo remote, may be obtained without producing all the terms.

NOTE. If the *ratio* of any geometrical ſeries be *double*, the *difference* of the *greateſt* and *leaſt* terms is equal to the ſum of all the terms, except the greateſt; if the ratio be *triple*, the *difference* is *double* the ſum of all but the greateſt; if the ratio be *quadruple*, the difference is *triple* the ſum of all but the greateſt, &c

In any ſeries in ∺ decreaſing to infinity—if the *ſquare* of the *firſt* term be divided by the *difference* between the *firſt* and *ſecond*, the quotient will be the ſum of the ſeries.

1, 2, 3, 4+ 4= 8
4, 16, 64, 256×256=65536=power of the ratio, whoſe exponent is leſs by 1, than the number of terms.

65536=8th power of the ratio.
Multiply by 4=firſt term.

262144=laſt term.

Or, $4\times4^{8}=262144$=the anſwer.

2. If the laſt term be 262144, the ratio 4, and the number of terms 9, what is the firſt term?

Laſt term.
8th power of the ratio $4^{8}=65536$)262144(4=firſt term.

Or, $\dfrac{262144}{4^{8}}=4$ the firſt term.

Again, Given the firſt term, and the ratio to find any other term aſſigned.

RULE I.

When the indices begin with an unit.

1. Write down a few of the leading terms of the ſeries, and place their indices over them.

2 Add together ſuch indices, whoſe ſum ſhall make up the entire index to the term required.

3. Multiply the terms of the geometrical ſeries, belonging to thoſe indices, together, and the product will be the term ſought.

EXAMPLES.

1. If the firſt term be 2 and the ratio 2, what is the 13th term?

1, 2, 3, 4, 5+ 5+3= 13
2, 4, 8, 16, 32×32×8=8192 Anſwer.

Or, $2\times2^{12}=8192$.

2. A merchant wanting to purchaſe a cargo of horſes for the Weſt-Indies, a jockey told him he would take all the trouble and expenſe, upon himſelf, of collecting and purchaſing 30 horſes for the voyage, if he would give him what the laſt horſe would come to by doubling the whole number by a half penny, that is, two farthings for the firſt, four for the ſecond, eight for the third, &c. to which the merchant, thinking he had made a very good

bargain, readily agreed: Pray, what did the laſt horſe come to, and what did the horſes, one with another, coſt him?

1, 2, 3, 4, 5, 6+ 6=12th 12+ 12+ 6=laſt ter.
2, 4, 8, 16, 32, 64×64=4096,&4096×4096×64=
1073741824qrs.=£1118481 1s. 4d. and their average price was £37282 14s. 0½d. a piece.

Rule II.

When the indices begin with a cypher.

1. Write down a few of the leading terms of the ſeries, as before, and place their indices over them.

2. Add together the moſt convenient indices to make an index, leſs by 1 than the number expreſſing the place of the term ſought.

3. Multiply the terms of the geometrical ſeries, together, belonging to thoſe indices, and make the product a dividend.

4. Raiſe the firſt term to a power, whoſe index is one leſs than the number of terms multiplied, and make the reſult a diviſor, by which divide the dividend, and the quotient will be that term beyond the firſt, ſignified by the ſum of thoſe indices, or the term ſought.

3. If the firſt term be 5 and the ratio 3, what is the 7th term?

0, 1, 2, 3+ 2+ 1= 6=index to 6th term beyond
5, 15, 45, 135×45×15=91125=dividend. [the 1ſt or 7th.

The number of terms multiplied is 3, (viz. 135×35×15) and 3—1=2 is the power to which the term 5 is to be raiſed; but the ſecond power of 5 is 5×5=25, and therefore 91125÷25=3645 the 7th term required.

Problem II.

Given the firſt term, the ratio, and number of terms, to find the ſum of the ſeries.

Rule. Raiſe the ratio to a power whoſe index ſhall be equal to the number of terms, from which ſubtract 1; divide the remainder by the ratio leſs 1. and the quotient, multiplied by the firſt term, will give the ſum of the ſeries.

EXAMPLES.

1. If the firſt term be 5, the ratio 3, and the number of terms 7, what is the ſum of the ſeries ?

Ratio=3×3×3×3×3×3×3=2187=7th power of
Subtract 1 [the ratio.

Divide by the ratio les 1=3—1=2)2186

Quotient=1093
Multiply by the firſt term= 5

Sum of the ſeries=5465

Or, $\frac{3^7-1}{3-1}\times5=5465$ Ans.

2. What debt can be diſcharged in a year, by paying 1 cent the firſt month, 10 cents the ſecond, and ſo on, each month in a tenfold proportion ?

$10^{12}-1\div10-1\times1=111111111111$c.=
D.1111111111 11c. Ans.

3. A man threſhed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the firſt day's work, 64 for the ſecond, and ſo on in an eight fold proportion ; I demand what his 9 days' labour amounted to, rating the wheat at 5s. per buſhel ?*

$\frac{8^9-1}{8-1}\times8=153391688$ corns. Amount=£78 0s. 5½d.

4. An ignorant fop wanting to purchaſe an elegant houſe, a facetious gentleman told him he had one which he would ſell him on theſe moderate terms, viz. that he ſhould give him a cent for the firſt door, 2 cents for the ſecond, 4 cents for the third, and ſo on, doubling at every door which were 36 in all ; it is a bargain, cried the ſimpleton, and here is a guinea to bind it : Pray what did the houſe coſt him ?

$\frac{2^{36}-1}{2-1}\times1=6871947673$5c.=D.687194767 35c. Ans.

---

* Note, 7680 wheat or barley corns are ſuppoſed to make a pint.

5. Suppoſe one farthing had been put out at 6 per cent. per annum, compound intereſt,[*] at the commencement of the Chriſtian æra: what would it have amounted to in 1784 years; and ſuppoſe the amount to be in ſtandard gold, allowing a cubick inch to be worth 53l. 2s. 8d. how large would the maſs have been?

$$\text{Ans. } \frac{2^{150}-1}{2-1} \times 1 =$$

£14867163465687482094357145515098907670685361 11s. 3¾d.

=27980859722121230415979571232933594210766 cubick inches of gold.

As 355 : 113 :: $\overline{360\times69{\cdot}5}$: 7964 earth's diameter.

360×69·5×7964×1327·33 = 264482820132 cubick miles in the globe.

=67273337308854741368832000 cub. inches in the globe.

Then, 27980859722121230415979571232933594210766
÷67273337308854741368832000

=415930899840288·8, which, however incredible it may appear to ſome, is more than four hundred and fifteen millions of millions, nine hundred and thirty thouſand eight hundred and ninety nine millions, eight hundred and forty thouſand two hundred and eighty eight times larger than the globe we inhabit.

PROBLEM III.

The firſt term, the laſt term, (or the extremes) and the ratio given, to find the ſum of the ſeries.

RULE. Divide the difference of the extremes by the ratio leſs by 1: Add the greater extreme to the quotient, and the reſult will be the ſum of all the terms.

Or, Multiply the greateſt term by the ratio, from the product ſubtract the leaſt term; then divide the remainder by the ratio, leſs by 1, and the quotient will be the ſum of all the terms.

[†] Any ſum, at 6l. per cent. per annum, compound intereſt, will double in 11 years and three hundred and twenty five days, or 11·889 years, or 11·89 is near enough; then if you divide 1784 by 11·89 it will give the number of terms in this caſe equal to 150:—The ratio will be 2, and the firſt term 1.

Or, When all the terms are given, then, from the product of the ſecond and laſt terms ſubtract the ſquare of the firſt term; this remainder being divided by the ſecond term leſs the firſt, will give the ſum of the ſeries.

EXAMPLES.

1. If the ſeries be 2, 6, 18, 54, 162, 486, 1458, 4374, what is its ſum total?

First Method.

From the greateſt term=4374
Subtract the leaſt = 2

Divide by the ratio—1=3—1=2)4372 diff. of extremes.

Quotient=2186
Add the greater extreme=4374

6560.

Or, $\frac{4374-2}{3-1}+4374=6560$ Ans.

Second Method.

Greateſt term=4374
Multiply by the ratio= 3

Product=13122
Subtract the leaſt term= 2

Divide by the ratio leſs by 1=3—1=2)13120

6560 Ans.

Or, $\frac{4374\times3-2}{3-1}=6560$.

Third Method.

Greateſt term=4374
Multiply by the ſecond term= 6

Product=26244
Subtract the ſquare of the firſt term=2×2= 4

Divide the remainder by the ſecond term leſs the firſt, } =6—2=4)26240

Ans. 6560

Or, $\frac{\overline{4374\times 6}-4}{6-2}=6560.$

2. A man travelled 6 days; the firſt day he went 4 miles, and each day doubling his day's travel, his laſt day's ride was 128 miles: How far did he go in the whole?

$$\frac{128-4}{2-1}+128=252 \text{ miles.}$$

3. A gentleman dying, left 5 ſons; to whom he bequeathed his eſtate as follows, viz. to his youngeſt ſon 1000l. to the eldeſt 5062l. 10s. and ordered that each ſon ſhould exceed the next younger by the equal ratio of $1\frac{1}{2}$: What did the ſeveral legacies amount to?

$$\frac{5062\cdot 5-1000}{1\cdot 5-1}+5062\cdot 5=£13187 \text{ 10s. Ans.}$$

PROBLEM IV.

Given the extremes and ratio, to find the number of terms.

R[illegible]E.

Divide the greateſt term [illegible] the leaſt; find what power of the ratio is equal to the quotient; then, add 1 to the index of that power, and the ſum will be the number of terms.

Or, Subtract the logarithm* of the leaſt term from that of the greateſt; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

EXAMPLES.

1. If the leaſt term be 2, the greateſt term 4374, and the ratio 3, what is the number of terms?

Divide by the leaſt term=2)4374=greateſt term.

$3\times 3\times 3\times 3\times 3\times 3\times 3$=quotient, 2187=7th power of the ratio; then 7+1=8 Ans.

---

* Logarithms are artificial numbers, the addition of which anſwers to multiplication of whole numbers, and ſubtraction to diviſion.

Or, from the logarithm of the greateſt term = 3·64088
Subtract the logarithm of the leaſt term = 0·30103
Divide the rema. by the logarithm of the ratio } = ·47712)3·33985(7+1=8, Anſwer.
3 33984

1

2. A gentleman travelled 252 miles; the firſt day he rode 4 miles, the laſt 128, and each day's journey was double to the preceding one: How many days was he in performing the journey? Ans. 6 days.

PROBLEM V.

Given the leaſt term, the ratio, and the ſum of the ſeries, to find the laſt term.

Rule. Multiply the ſum of the ſeries by the ratio, leſs 1; to that product add the firſt term, and the reſult, divided by the ratio, will give the laſt term.

EXAMPLES.

If the 1ſt term be 2, the ratio 3, and the ſum of the ſeries 6560; what is the laſt term?

Sum of the ſeries=6560
Multiply b[illegible]he ratio leſs 1=, 2

Product=13120
Add the leaſt term= 2

Divide their ſum by the ratio=3)13122

4374 Ans.

Or, $\frac{\overline{3-1}\times 6560+2}{3}=4374$ Ans.

2. A gentleman performed a journey of 252 miles; the firſt day he rode 4 miles, and each day after the firſt, twice ſo far as the day before: How far did he ride the laſt day?

$\frac{\overline{2-1}\times 252+4}{2}=128$ miles, Anſwer.

SIMPLE INTEREST.

INTEREST is a premium allowed by the borrower to the lender, according to a certain rate per cent. agreed on; which by law is ſtated at 6 per cent. per annum.— *Principal* is the money lent. *Rate* is the ſum per cent. agreed on. *Amount* is the ſum of principal and intereſt.

Simple Intereſt is that, which is allowed on the principal only.

Note. By this rule, Commiſſion, Brokerage, Inſurance, Purchaſing Stocks, or any thing elſe, rated at ſo much per cent. are calculated.

General Rule.

1. Multiply the principal by the rate, and divide by 100, (or cut off the two right hand figures in the pounds) and the quotient, or left hand figures, will be the anſwer in pounds, &c. the right hand figures being reduced and cut off as at firſt. If the principal be dollars, the right hand figures will be cents.

2. For more years than one, multiply the intereſt of one year by the number of years.

3. For any number of months, take the aliquot parts of a year; and for days the aliquot parts of 30.

Note. When the rate per cent. per annum is

	multiply the principal by
9	$\frac{3}{4}$
8	$\frac{2}{3}$
6	$\frac{1}{2}$
4	$\frac{1}{3}$
3	$\frac{1}{4}$
2	$\frac{1}{6}$

of the given number of months, and you will have the intereſt for the given time.

Examples.

1. What is the intereſt of 573l. 13s. 9½d. at 6 per cent. per annum? Ans. £34 8s. 5d.

```
£573   13   9½
                6
£34|42   2   9
      20
    8|42
      12
    5|13
       4
    1|52
```

2. What is the intereſt of £329 17s. 6½d. for 3 years, 7 months, and 12 days, at 5 per cent. per annum?

£329 17 6½
5

16|49 7 8½
20

9|87
12

10|52
4

2|10

Then,

6 mo.	½	16	9	10½	intereſt of 1 year.
				3	
		49	9	7½	do. of 3 years.
1 mo.	1/6	8	4	11¼	do. of 6 months.
10 da.	⅓	1	7	5¾	do. of 1 month.
2 da.	⅕		9	1¾	do. of 10 days.
			1	9¾	do. of 2 days.
	Ans.	£59	13		do. of 3y. 7m. 12d.

Or thus:

		£	s.	d.
£5	1/10	329	17	6½
6 months	½	16	9	10½
				3
		49	9	7½
1 month	1/6	8	4	11¼
10 days	⅓	1	7	5¾
2 days	¼		9	1¾
			1	9¾
		£59	13s.	Ans.

3. What is the intereſt of 347 dollars 50 cents, at 6 per cent. per annum for a year?

D347·50

6

20·8500 Ans. D20 85c.

4. What is the intereſt of D797 13c. at 6 per cent. per annum, for 8 months?

D797·13

4

31·8852 Ans. D31 88c. 5$\frac{2}{10}$m.

5. What is the intereſt of D649 17c. at 6 per cent. per annum, for 15 months?

D649·17

7$\frac{1}{2}$

454419

324585

48·68775 Ans. D48 68c. 7$\frac{3}{4}$m.

6. Required the amount of £725 12s. 6d. at 5 per cent. per annum, for a year.

5=$\frac{1}{20}$)725 12 6

36 5 7$\frac{1}{2}$

Ans. £761 18 1$\frac{1}{2}$

7. What is the amount of D560 50c. at 6 per cent. for 16 months?

D560 50

8

44·84

560·50

Ans. D605·34

8. What is the intereſt of D150 75c. for 1 month, at 6 per cent. per annum?

```
½)150·75
  ------
   ·75375   Ans. 75c. 3¾m.
```

So that any number of dollars, conſidered as ſo many cents, is the intereſt for 2 months, at 6 per cent.

COMMISSION, OR FACTORAGE,

Is an allowance of ſo much per cent. to a Factor or Correſpondent, for buying and ſelling goods.

9. Required the commiſſion on £436 9s. 6d. at 3½ per cent.

```
£436   9   6
           3½
-------------
 1309   8   6
  218   4   9
-------------
 15|27 13   3
    20
  -----
  5|53
    12
  -----
  6|39
     4
  -----
  1|56 Ans. £15 5 6¼.
```

10. Required the commiſſion on D649 75c. at 1¾ per cent.

```
| ½ | 649·75
| ¼ | 324·875
      162·4375
  -------------
  1137·0625 Ans. D11 37c.0⅝m.
```

BROKERAGE,

Is an allowance of ſo much per cent. to a perſon called a Broker, for aſſiſting merchants in purchaſing or ſelling goods.

11. Required the brokerage on £911 12s. at 5s. or $\frac{1}{4}$ per cent.

5s.=$\frac{1}{4}$)911 12
2|27 18
20
5|58
12
6|96
4
3|84

Ans. £2 5s. 6$\frac{3}{4}$d.

12. Required the brokerage on D876 21c. at 33$\frac{1}{3}$ cents, or at $\frac{1}{3}$ per cent.

$\frac{1}{3}$)876·21
292·07 Ans. D2 92c. 0$\frac{7}{10}$m.

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies.

13. Required the amount of £375 15s. bank ſtock, at £75 per cent.

| 50 | $\frac{1}{2}$ | 375 15
| 25 | $\frac{1}{2}$ | 187 17 6
93 18 9
Ans. £281 16 3

Or thus:
| 25 | $\frac{1}{4}$ | 375 15
Subtract 93 18 9
As before, £281 16 3

14. Required the amount of D2195 50c. bank ſtock, at 125 per cent.

| 25 | $\frac{1}{4}$ | 2195·50
Add 548·875
Ans. D2744·375

SIMPLE INTEREST BY DECIMALS.

A TABLE of Ratios, from *one* pound to *ten* pounds.

Rate per cent.	Ratios.	Rate pr.ct.	Ratios.	Rate pr.ct.	Ratios.
1	·01	3	·03	5	·05
1¼	·0125	3¼	·0325	5½	·055
1½	·015	3½	·035	6	·06
1¾	·0175	3¾	·0375	6½	·065
2	·02	4	·04	7	·07
2¼	·0225	4¼	·0425	8	·08
2½	·025	4½	·045	9	·09
2¾	·0275	4¾	·0475	10	·1

Ratio is the Simple Intereſt of £1 or Dollar, for 1 year, at the rate per cent. agreed on.

A TABLE for the ready finding of the decimal parts of a year, equal to any number of *days*, or *quarters* of a year.

Days	dec. pts.	Days.	dec. pts.	Days	dec. pts.
1	·00274	10	·027397	100	·273973
2	·005479	20	·054794	200	·547945
3	·008219	30	·082192	300	·821918
4	·010959	40	·109589	400	1·000000
5	·013699	50	·136986		
6	·016438	60	·164383	¼ of a year=·25	
7	·019178	70	·191781	½ of a year=·5	
8	·021918	80	·219178	¾ of a year=·75	
9	·024657	90	·246575		

CASE I.

The principal, time, and ratio given, to find the intereſt and amount.

Rule. Multiply the principal, time, and ratio continually together, and the laſt product will be the intereſt, commiſſion, brokerage, &c. to which add the principal, and the ſum will be the amount.

Examples.

1. Required the amount of £537 10s. at £6 per cent. per annum, for 5 years?

Principal 537·5
Multiply by the ratio= ·06

Product 32·250
Multiply by the time= 5

Intereſt=161·250
Add the principal=537·5

Amount=£698·75
20

15·00 Ans. £698 15s.

Or, 537·5×·06×5+537·5=£698 15s.

2. What is the ſimple intereſt of £917 16s. at £5 per cent. per annum, for 7 years? Ans. £321 4s. 7d.

3. What is the amount of £235 3s. 9d. at £5¼ per cent. per annum, from March 5th, 1784, to November 23d, 1784? Ans. £244 0 8½.

4. If my correſpondent is to have £2½ per cent. what will his commiſſion on £785 15s. amount to?
Ans. £19 12 10½.

5. If a broker diſpoſes of a cargo for me, to the amount of £637 10s. on commiſſion at £1¼ per cent. and procures me another cargo of the value of £817 15s. on commiſſion at £1¾ per cent.; what will his commiſſion on both cargoes amount to? Ans £22 5 7.

CASE II.

The amount, time, and ratio given, to find the principal.

Rule.

Multiply the ratio by the time; add unity to the product, for a diviſor, by which ſum divide the amount, and the quotient will be the principal.

EXAMPLES.

1. What principal will amount to £1045 14s. in 7 years, at £6 per cent. per annum?

Ratio = ·06
Multiply by the time = 7

Product = ·42
Add 1·

Diviſor = 1·42)1045·7(736·4084 + = £736 8s. 2d. anſwer.

Or, $\frac{1045 \cdot 7}{\cdot 06 \times 7 + 1}$ = £736 8s. 2d. Ans.

2. What principal will amount to £3810 in 6 years, at £4½ per cent. per annum? Ans. £3000.

3 What principal will amount to £666 9s. 0¼d. in 3½ years, at £5¼ per cent. per annum? Ans. £563.

CASE III.

The amount, principal, and time given, to find the ratio.

RULE.

Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

EXAMPLES.

1. At what rate per cent. will £543 amount to £705 18s. in 5 years?

From the amount = 705·9
Take the principal = 543

Divide by 543×5 = 2715)162·90(·06
162·90

Or, $\frac{705 \cdot 9 - 543}{543 \times 5}$ = ·06 = £6 Anſwer.

2. At what rate per cent. will £391 17s. amount to £449 3s. 1¼d. ·744qr. in 3¼ years? Ans. £4½.

3. At what rate per cent. will £413 12s. 6d. amount to £546 4s. 10¼d. in 4¾ years? Ans. £6¾.

CASE IV.

The amount, principal, and rate per cent. given, to find the time.

Rule.

Subtract the principal from the amount; divide the remainder by the product of the ratio and principal, and the quotient will be the time.

Examples.

1. In what time will £543 amount to £705 18s. at £6 per cent. per annum?

From the amount=705·9
Take the principal=543

Divide by 543×·06=32·58)162·9(5 years, Ans.
162 9

2. In what time will £3000 amount to £3810, at £4½ per cent. per annum? Ans. 6 years.

3. In what time will £391 17s. amount to £441 3s. 1¾d. at £4¼ per cent. per annum? Ans. 3¼ years.

To find the interest of any sum at 6 per cent. per annum, for any number of months.

Rule.

If the months be an even number, multiply the principal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus the half of 19 is 9·5; and multiply the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What is the interest of £345 16s. 6d. for 9 years and 11 months, at 6 per cent. per annum?

Y. m.
9 11
12

2)119 months.

59·5=½ number of months.

Principal 345·825
Mult. by ½ number of Mo. 59·5

1729125
3112425
1729125

£205·765875=£205 15 3¾ Ans.
Principal=£345 16 6

Amount=£551 11 9¾

A TABLE of decimal parts for every day in the twelfth part of a year, which conſiſts of 365¼ days.

D	dec.	D.	dec	D	dec.	D.	dec.	D.	dec.
1	033	7	·230	13	·427	19	·624	25	·821
2	·066	8	·263	14	460	20	·657	26	·854
3	·098	9	·296	15	493	21	·690	27	·887
4	·131	10	·328	16	526	22	·723	28	·920
5	·164	11	·361	17	·558	23	·756	29	·953
[illegible]	[illegible]	12	·394	18	·591	24	·788	30	·986

Note. This Table may alſo be uſed for the *parts of a year*, in Compound Intereſt after having worked for whole years.

To find the Intereſt of any ſum, either for months, or months and days, at 6 per cent. per annum.

Rule.

Multiply the principal by the number of months, (or months and parts anſwering to the given number of days in the table); and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the intereſt for the given time, in ſhillings and decimal parts of a ſhilling.

Examples.

1. What is the intereſt of £100 for a year?

Principal=100
Multiply by the months=12

Ans. s.120|0=£6

2. What is the intereſt of £250 10s. for 19 months and 7 days?

```
          Principal=£250·5
             Time= 19·23
                  ------
                    7515
                   5010
                  22545
                  2505
                 -------
     Ans. s.481·7115=£24 1 8½
```

Another method of calculating Intereſt for Months, at £6 per cent. per annum.

RULE.

If the principal conſiſt of pounds only, cut off the unit figure, and, as it then ſtands, it will be the intereſt for 1 month in ſhillings and decimal parts:—If it conſiſt of pounds, ſhillings, &c. reduce the ſhillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a ſhilling.

EXAMPLES.

1. What is the intereſt of £175 for 5 months?

```
£175=17·5s.=intereſt for 1
month.                 17·5
Multiply by the time=     5
                     ------
                 2|0)87·5
                     ------
              Ans £4 7 6
```

2. What is the intereſt of 255l. 16s. for 7 months?

```
              ſhil.    [month.
£255 16=25·58 int. for 1
              7
          ------
   2|0)179·06
          ------
   £8 19 1½ Ans.
```

SIMPLE INTEREST IN FEDERAL MONEY.

PROBLEM I.

When the principal is given in Maſſachuſetts pounds, ſhillings, &c and the intereſt is required in Federal Money, at 6 per cent. per annum.

RULE.

Reduce the ſhillings, &c. to their equivalent decimal, divide the whole by 5, and the quotient is the annual intereſt: Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off,) will be the intereſt as before.

EXAMPLES.

1. Required the annual intereſt of 517l. 3s. 7$\frac{1}{8}$d. at 6 per cent.

3s.=·15 5)517·181

7$\frac{1}{8}$d.=·030 D. c. m.

Exceſs of 12 =·001 103·416=103 43 6 Ans.

·181 Or, 517·181

2

103·4362=D103 43c. 6$\frac{2}{10}$m.

2. Required the annual intereſt of 1l. in cents.

5)1·00

20 cents, Ans.

PROBLEM II.

When the principal is given in Maſſachuſetts old currency, and the intereſt and amount are required in Federal Money at 6 per cent.

RULE.

Reduce the Maſſachuſetts money to Federal, then divide the principal by 20, and that quotient by 5; add thoſe quotients together, and they are the intereſt; or add them to the principal, and they are the amount.

EXAMPLES.

1. Required the amount of 425l. 16s. 8$\frac{1}{2}$d. for 1 year, at 6 per cent.

·8 3)425·835

·034 20)1419·450

·001 5)70·9725

14·1945

16s. 8d.=£.835 D. c. m.

1504·6170=1504 61 7 Ans.

2. Required the amount of £112 4s. 6d. for 1 year.

·2 3)112·225
·034 20)374·083
·001 5)18·7041
3·7408
4s. 6d.=£·225 D. c. m. dec.
396·52 79=396·52 7 9 Ans.

PROBLEM III.

When the principal is Maſſachuſetts old currency, and the monthly intereſt is required in Federal Money.

RULE.

Reduce the ſhillings, &c. to decimals, then ſeparate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the anſwer in dollars, cents, &c.

EXAMPLE.

Required the monthly intereſt of 425l. 16s. 8$\frac{1}{2}$d. in Federal money?

·8 6)42·5835
·034 D. c. m.
·001 7·09725=7·09 7$\frac{1}{4}$ Ans.
£·835

PROBLEM IV.

When the principal is Federal Money, and the intereſt is required in the ſame.

RULE.

Work according to the general rule in ſimple intereſt; that is, multiply by the rate of intereſt, ſeparate the two right hand figures of the dollars in the product, and it will give the intereſt in dollars, cents, &c.

N B. The figures which are more than three places to the right hand of the point, are of no account, unleſs the fourth place exceed 5, in which caſe increaſe the mills 1.

EXAMPLES.

1. What is the annual intereſt of D537 24c. 6m. at 6 per cent.?

537·24 6
6
D. c. m.
32·23476=32·23 5 Ans.

2. What is the intereſt of D1465 46c. 6m. for 16 months, at 6 per cent. per annum?

1465·46 6
8=half the number of months.
——— D. c. m.
117·23728=117·23 7 Ans.

3. What is the intereſt of D537 34c. 7m. for 19 months, at 6 per cent. per annum?

537·34 7
9·5=half the number of months.
———
2686735
4836123
——— D. c. m.
51·047965=51·04 8 Ans.

N. B. Becauſe there are 4 decimals in the multiplicand and multiplier, I cut off 4 figures for them, and two more according to the rule.

PROBLEM V.

When the principal is Federal Money, and the monthly intereſt is required in the ſame, at 6 per cent. per annum.

RULE.

Separate the two right hand figures of the dollars, and you then have the intereſt for two months; half of which is the monthly intereſt in dollars, cents, &c. If there be but one place, or figure of dollars, a cypher muſt be prefixed to the left hand.

EXAMPLES.

1. What is the monthly intereſt of D9 59c. 7m. at 6 per cent. per annum? 2)·09597
Ans. ·047985=4c. 8m. nearly.

2. What is the monthly intereſt of D100 50c. 5m.?
2)1·00505
·50252=50c. 2½m. Ans.

RULES *for calculating Intereſt for Days.*

RULE I.

Multiply the principal by the given number of days, and that product by the rate on the pound; divide the laſt product by 365 (the number of days in a year) and it will give the intereſt.

EXAMPLE.

What is the intereſt of 360l. 10s. for 175 days, at 6 per cent.?

$$\frac{360\cdot5\times175\times\cdot06}{365}=\text{£}10\cdot37=\text{£}10\ 7\text{s.}\ 4\tfrac{3}{4}\text{d. Anſwer.}$$

RULE II.

Multiply the given principal by the given number of days, and divide the product by 6083, for 6 per cent.; (the number of days in which any ſum will double at that rate) the quotient will give the anſwer.

EXAMPLE.

What is the intereſt of 327l. 10s. at 6 per cent. per annum, for 210 days?

$$\frac{327\cdot5\times210}{6083}=\text{£}11\cdot306=\text{£}11\ 6\ 1\tfrac{1}{4}\ \text{Ans.}$$

RULE *for making a diviſor for any rate per cent.*

Multiply 365 by 100, and divide the product by the rate.

Thus for 6 per cent. $\frac{365\times100}{6}=6083$ diviſor.

For 5 per cent. $\frac{365\times100}{5}=7300$ diviſor, &c.

Perhaps the moſt convenient way to calculate at 6 per cent is firſt to do it for 5, and then add one fifth of the quotient to itſelf; becauſe, by cutting off the two cyphers in the diviſor, you have to divide only by 73.

Hence, when intereſt is to be calculated on caſh accounts, accounts current, or any other accounts where partial payments are made, or partial debts contracted; multiply the ſeveral balances into the days they are at intereſt and the ſum of theſe products, divided as above, will give the intereſt at 5l. or 6l. per cent. and for any other rate, make the proper addition or deduction; or find a diviſor, as before directed.

Examples.

1. On the 1ſt of January I lent 450l. 10s. 6d. which I received back in the following partial payments, viz. on the 14th of Jan. 57l. 11s. 9d.; on the 7th of February, 39l. 3s. 10d.; on the 19th of March, 63l. 5s. 2d.; on the 4th of April, 45l.; on the 26th of April, 19l. 12s. 6d.; on the 12th of May, 100l.; on the 10th of June, 60l. 7s. 3d.; and on the 1ſt of Auguſt, 65l. 10s: What intereſt is due at 6 per cent.?

Dates.		£	s.	d.	Da.	Products.		
Jan. 1	Lent on demand	450	10	6	13	5856	16	6
14	Received in part	55	11	9				
	Balance	392	18	9	24	9430	10	0
Feb. 7	Received in part	39	3	10				
	Balance	353	14	11	40	14149	16	8
Mar. 19	Received in part	63	5	2				
	Balance	290	9	9	16	4647	16	0
April 4	Received in part	45	0	0				
	Balance	245	9	9	22	5400	14	6
26	Received in part	19	12	6				
	Balance	225	17	3	16	3613	16	0
May [illegible]	Received in part	100	0	0				
	Balance	125	17	3	29	3650	0	3
June 10	Received in part	60	7	3				
	Balance	65	10	0	52	3406	0	0
Aug. 1	Received in full of the principal	65	10	0		50155	9	11

5)
73|00)501|55 9 11(6 17 4¾ intereſt at 5 per cent.
438 1 7 5¾
6355 £8 4 10½ intereſt at 6 per cent.
20
)1271|09
73
541
511
3009
12
)361|19
292
6919
4
)276|76
219
5776

2. I have given Peter Truſty a caſh credit for 1000l. in conſequence of which, on the 12th of May, I paid his bill for 250l.; May 27th, paid his draught for 280l.; June 1ſt, he gave me a bill on the Maſſachuſetts bank at ſight, for 290l.; July 17th, he paid me per receipt, 70l.; Auguſt 20th, he drew for 750l. at ſight; Auguſt 31, he paid me per receipt, 500l.; Sept. 15th, he drew at ſight for 135l.; and 3d of October for 175l.; Oct. 29th, he paid me, per receipt, 250l.; and Nov. 3d, 125l.; November 12th, he drew at ſight for 375l.; and November 18th, for 125l.; January 1ſt, he paid me, per receipt, 290l.; and January 20th, 210l. On the 1ſt of March following, he demands a ſettlement: What is then due to me, intereſt at 6 per cent.?

Dates.				£	Days.	Prod.
May	12	Paid his bill		250	15	3750
May	27	Paid his draught		280		
			Balance	530	5	2650
June	1	Received in part		290		
			Balance	240	46	11040
July	17	Received in part		70		
			Balance	170	34	5780
Auguſt	20	Paid		750		
			Balance	920	11	10120
Auguſt	31	Received in part		500		
			Balance	420	15	6300
Sept.	15	Paid		135		
			Balance	555	18	9990
October	3	Paid		175		
			Balance	730	26	18980
October	29	Received in part		250		
			Balance	480	5	2400
November	3	Received in part		125		
			Balance	355	9	3195
Nov.	12	Paid		375		
			Balance	730	6	4380
Nov.	18	Paid		125		
			Balance	855	44	37620
January	1	Received in part		290		
			Balance	565	19	10735
January	20	Received in part		210		
March	1		Balance	355	40	14200
						141140

```
                    5)
Then, 73|00)141140( 19   6   8 intereſt at 5 per cent.
                     3  17   4
                    ————————————
                   £23   4   0 intereſt at 6 per cent.
                   355   0   0
                    ————————————
                  £378   4   0 balance in my favour.
```

When caſh credits are given, a balance ſhould be made upon every tranſaction, which ſhould be multiplied into the days the firſt leiſure minute; then, when the time of ſettlement comes, you will only have to add up the products, and divide as above, and the account will be finiſhed.

3. A owes B the following ſums, with the intereſt on them, at 6 per cent. per annum, as follows, viz. D60 for 7 months, D150 for 15 months, D75 50c. for 9 months, D145 75c. for 27 months, and D397 60c. for 45½ months: What is the amount of principal and intereſt?

```
 D.c.   Mo.
 60     × 7    =   420
150     ×15    =  2250
 75·5   × 9    =   679·5
145·75  ×27    =  3935·25
397·60  ×45·5  = 18090·8
————————          ——————
828·85     200)25375·55(126·877 intereſt.
                        828·85  principal.
                       ————————
                      D955·727 amount, Ans.
```

Note. I divide by 200, the number of months, in which any ſum will double at 6 per cent. per annum, and it gives the intereſt.

When partial payments are made upon bonds, notes, &c. at any interval *greater* than a year, the intereſt is calculated in a progreſſive manner, by adding the intereſt to the principal at the time of the firſt payment, and from the ſum deducting the payment, &c.

DISCOUNT

IS an allowance made for the payment of any ſum of money, before it becomes due, and is the difference between that ſum, due ſome time hence, and its preſent worth.

The *preſent worth* of any ſum or debt, due ſome time hence, is ſuch a ſum, as, if put to intereſt, would in that time and at the rate per cent. for which the diſcount is to made, amount to the ſum or debt then due.

RULE.

As the amount of 100l. for the given rate and time, is to 100l. : ſo is the given ſum or debt to the preſent worth.

Subtract the preſent worth from the given ſum, and the remainder will be the diſcount required.

Or, As the amount of 100l. for the given rate and time, is to the intereſt of 100l. for that time : ſo is the given ſum or debt to the diſcount required.

Or, *In Federal Money*, divide the given ſum by the amount of D100 for the given time and rate; point off from the right of the quotient two places leſs than in diviſion of decimals, for the preſent worth.

EXAMPLES.

1. What is the diſcount of $\left\{\begin{array}{l}\text{£635 17s.}\\ \text{D2119 50c.}\end{array}\right\}$ due two years hence, at 5½ per cent. per annum?

Intereſt of 100l. per annum=5 10
2 years.

11
Add 100

111

As £111 : £11 :: 635l. 17s. : 63l. 0s. 2¾d. diſc. Anſ.

Or,

	£		£		£	s.		£	s.	d.	
As	111	:	100	::	635	17	:	572	16	9¼	preſent worth.

And 635l. 17s.—572l. 16s. 9¼d.=63l. 0s. 2¾d. diſcount.

In Federal Money.

D. D. D. c. D. c. m.

As 111 : 11 :: 2119 50 : 210 04 0½=diſcount. Or,

As 111 : 100 :: 2119 50 : $\frac{2119{\cdot}5\times100}{111}$ =D1909 45c. 9½m=preſent worth; and 2119·5—1909·4595=210·0405 =diſcount, as before.

Or, $\frac{2119{\cdot}5}{111}$ = 19·094595; and 1909·4095 = preſent worth as before.

2. What is the preſent worth of D350 payable in half a year, diſcounting at 6 per cent. per annum?

Ans. D339 80c. 5m.

3. What is the preſent worth of 65l. due 15 months hence, at 6l. per cent. per annum? Ans. 60l. 9s. 3½d.

4. What is the diſcount on 97l. 10s. due January 22, this being Sept. 7, reckoning intereſt at 5l. per cent.?

Ans. 1l. 15s. 11d.

5. Bought a quantity of goods for D250 ready money, and ſold them for D300 payable 9 months hence: What was the gain in ready money, ſuppoſing diſcount to be made at 6 per cent.? Ans. D37 8c. 1¼m.

BARTER

IS the exchanging of one commodity for another, and teaches traders to proportion their quantities without loſs.

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1lb. 1cwt 1yd &c. as alſo the value of the integer of ſome other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of ſelling it.

Rule.

Find the value of the given quantity by the conciſeſt method, then find what quantity of the other, at the rate propoſed, you may have for the ſame money: *Or*, if the quantity be given, find from thence the rate of ſelling it. *Or*, As the quantity of one article is to its price, ſo, *inverſely*, is the quantity of the other to its price. *Or*, as the price of one article is to its quantity; ſo *inverſely*, is the price of the other to its quantity.

EXAMPLES.

1. How much tea, at 9s. 6d. per lb. muſt be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

		Gals.
3d.	$\frac{1}{4}$	156
$\frac{1}{3}$	$\frac{1}{6}$	12
		1872
		39
		6 6
9s. 6d.=114d.		1917 6
		12
		23010

d. lb. d. lb. oz.
As 114 : 1 :: 23010 : 201 $13\frac{54}{114}$ Ans.

price. quan. price. quan.
Or, As 12s $3\frac{1}{2}$d. : 156gal. :: 9s. 6. : 201lb. $13\frac{54}{114}$oz. anſwer, as before.

2. How much cloth, at 15s. 8d. per yard, muſt be given for 5cwt. 3qrs. 19lb. of ſteel, at 5 guineas per cwt.?
Ans. 52yds. 3qrs. 2n.

3. Suppoſe A has 350 yards of linen, at 25c. per yard, which he would truck with B for ſugar, at D5 per cwt. How much will the linen come to?
Ans. 17cwt. 2qrs.

4. A has broadcloths at D44 per piece, and B has mace, at D1 42c. per lb: How many pounds of mace muſt B give A for 35 pieces of cloth? Ans. $1804\frac{1}{8}$lbs.

5. A has $7\frac{1}{2}$cwt. of ſugar at 12 cents per lb. for which B gave him $12\frac{1}{2}$cwt. of flour: What was the flour rated at per lb.?
Ans. 7c. 2m.

CASE II.

If the quantities of two commodities be given, and the rate of ſelling them, to find, in caſe of inequality, how much of ſome other commodity muſt be given.

RULE.

Find the ſeparate value of the two given commodities; ſubtract the leſs from the greater, and the difference will be the amount of the third commodity, whoſe quality and rate may be eaſily found.

EXAMPLES.

1. Two merchants barter; A has 30cwt. of cheeſe, at 23s. 6d. per cwt. and B has 9 pieces of broadcloth, at 3l. 15s. per piece: Which muſt receive money, and how much? Ans. B muſt pay A 1l. 10s.

2. A and B would barter; A has 150 buſhels of wheat, at D1 25c. per buſhel, for which B gives 65 buſhels of barley, worth $62\frac{1}{2}$c per buſhel, and the balance in oats at $37\frac{1}{2}$c. per buſhel: What quantity of oats muſt A receive from B? Ans. $391\frac{2}{3}$ buſhels.

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, ſay,

As the ready money price of the one, is to its bartering price; ſo is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready money price.

EXAMPLES.

1. A has ribbands at 2s. per yard ready money; but in barter he will have 2s. 3d.; B has broadcloth at 32s. 6d. per yard ready money; at what rate muſt B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2s. 3d. per yard, muſt then be given by A for 488 yards of B's broadcloth?

Ans. B's broadcloth, at £1 16s. $6\frac{1}{2}$d. per yd.; 7930 yds. ribband.

2. A and B barter; A has 150 gallons of brandy, at D1 $37\frac{1}{2}$c per gallon ready money, but in barter he will have D1 50c.; B has linen at 44c per yard ready money; how muſt B ſell his linen per yard in proportion to A's

bartering price, and how many yards are equal to A's brandy?

Ans. Barter price is 48c. and he muſt give A $468\frac{3}{4}$yds.

3. P and Q barter: P has Iriſh linen, at 60c. per yard, but in barter he will have 64c. Q delivers him broadcloth at D6 per yard, worth only D5 50c. per yard: Pray which had the advantage in barter, and how much linen does P give Q for 148 yards of broadcloth?

c. c. D c. D c.
As 60 : 64 :: 5 50 : 5 $86\frac{2}{3}$; therefore, Q, by ſelling at D6, has the advantage. Then,

D yds. c. yds. qrs.
As 6 : 148 :: 64 : 1387 2 linen, Ans.

4. A has 200 yards of linen, at 1s 6d. ready money per yard, which he barters with B, at 1s. 9d per yard, taking buttons at $7\frac{1}{2}$d. per groſs, which are worth but 6d.: How many groſs of buttons will pay for the linen, who gets the beſt bargain, and by how much, both in the whole, and per cent.?

yd. d. yds. d. d. groſs. d. groſs.
As 1 : 21 :: 200 : 4200. As $7\frac{1}{2}$: 1 :: 4200 : 560

yd. d. yds. £
As 1 : 18 :: 200 15 value of A's linen.

gr. d. gr. £
As 1 : 6 :: 560 : 14 value of B's goods. So that B gains 1l. of A.

£ £ £
As 14 : 1 : 100 : 7l. 2s. 10d. per cent.

LOSS AND GAIN

IS an excellent rule, by which merchants and traders diſcover their profit, or loſs per cent. or by the groſs: It alſo inſtructs them to raiſe or fall the price of their goods, ſo as to gain or loſe ſo much per cent. &c.

CASE I.

To know what is gained or loſt per cent.

RULE

Firſt ſee what the gain or loſs is, by ſubtraction; then, as the price it coſt, is to the gain or loſs: ſo is 100l. to the gain or loſs per cent. &c.

Or, in *federal money*, annex two cyphers to the gain or loſs, and divide by the coſt for the gain or loſs per cent.

EXAMPLES.

1. If I buy ſerge at 90c. per yard, and ſell it again at D1 2c. per yard: What do I gain per cent. or in laying out D100?

			c.		c.		D.		D.	
Sold for	D1·02	As	90	:	12	::	100	:	13⅓	per cent. gain, Ans.
Coſt	·90									
Gain	·12 per yard.									

Or, 1·02 − ·90 = ·12 = gain per yard; and $\frac{12\cdot00}{\cdot9}$ = 13⅓ per cent. gain, as before.

N. B. The firſt queſtions in the ſeveral caſes ſerve to elucidate each other.

2. If I buy ſerge at D1 2c. per yard, and ſell it again at 90c.: What do I loſe per cent. or in laying out D100?

			D.		c.		D.		D. c. m.	
Coſt	D1·02									
Sold for	·90	As	1·02	:	12	:	100	:	11 76 5	per cent. loſs, Ans.
Loſs	·12									

Or, $\frac{12\cdot00}{1\cdot02}$ = 11·215 per cent. loſs, Ans. as before.

3. If I buy a cwt. of tobacco for 9l. 6s. 8d. and ſell it again at 1s. 10d. per lb. do I gain or loſe, and what per cent.?

	£	s.	d.	
Sold for	10	5	4	
Coſt	9	6	8	
	0	18	8	gained in the groſs.

		£	s.		
2d	$\frac{1}{12}$	11	4		value at 2s. per lb.
		0	18	8	value at 2d. per lb.
		10	5	4	value at 1s. 10d. per lb.

	£	s.	d.		£	s.		£		£	
As	9	6	8	:	18	8	::	100	:	10	Ans. 10 per cent. gain.

4. A draper bought 60 yards of cloth at D4 50c. per yard, and 38 yards of cloth at D2 50c. per yard, and ſold them, at D4 25c. per yard: Did he gain or loſe, and what per cent.?

60 yards at D4 50c. per yard = D270
38 yards at 2 50 per yard = 95

98 yards coſt 365
which ſubtract from 98 yards at D4 25c. = 416·50

Gained in the groſs, = 51·50

D. D. c. D. D. c.
Then, as 365 : 51·50 :: 100 : $\frac{5150·00}{365}$ = 14·11 gain per cent. Ans.

5. Bought ſugar at 6½d. per lb. and ſold it at 2l. 3s. 9d. per cwt.: What was the gain or loſs per cent.?

lb. d. lb. £ s. d.
As 1 : 6½ :: 112 : 3 0 8
Prime coſt £3 0 8 per cwt.
Sold at 2 3 9 per cwt.

Loſt £0 16 11 in the whole.

£ s. d. £ s. £ £ s. d.
As 3 0 8 : 16 11 :: 100 : 27 17 8¼ loſs per cent. Ans.

6. At 4s. 6d. in the pound profit, how much per cent.?

£ s. d. £ £ s.
As 1 : 4 6 :: 100 : 22 10 Ans.

7. If I buy candles at 1s. 6d. per lb. and ſell them again at 2s. per lb. and allow 3 months for payment, what do I gain per cent.?

d. d. £ £ s. d.
As 18 : 24 :: 100 : 133 6 8;

Mo. £ Mo. £
Then, by Diſcount, As 12 : 6 :: 3 : 1 10s.

£ s. £ s. £ s. d. £ s. d.
Then, As 101 10 : 1 10 :: 133 6 8 : 1 19 4¾
which taken from 133l. 6s. 8d. leaves 131l. 7s. 3¼d. therefore, Ans. 31l. 7s. 3¼d.

6. If I buy cloth at 13s. per yard, on 8 months credit, and ſell it again at 12s. ready money, do I gain or loſe, and what per cent.?

Mo. £ Mo. £ £ s. £ s. d.
As 12 : 6 :: 8 : 4 As 104 : 13 :: 100 : 12 6;

So that 13s. on 8 months credit, at 6 per cent. is equal to 12s. 6d. ready money; then,

Prime coſt 12s. 6d. ready money.
Sold at 12 0 ready money.

Loſt 6 in the yard.
As 12s. 6d. : 6d. :: £100 : £4 Ans. 4l. per cent.

CASE II.

To know how a commodity muſt be ſold, to gain or loſe ſo much per cent.

RULE. As 100l. is to the price; ſo is 100l. with the profit added, or loſs ſubtracted, to the gaining or loſing price.

Or, In *Federal Money*, multiply 100 dollars added to the gain, or leſs by the loſs per cent. by the coſt; and pointing off the two right hand figures of the product gives the anſwer.

EXAMPLES.

1. If I buy a quantity of ſerge, at 90c. per yard: How muſt I ſell it per yard to gain $13\frac{1}{3}$ per cent.?

D. D. c. c. D. c.
As 100 : 113 $33\frac{1}{3}$ :: 90 : 1 2 Ans.

Or, D113 $33\frac{1}{3}$c.×90c.=D102; and pointing off two right hand places, D1·02, Ans. as before.

2. If a barrel of powder coſt 4l. how muſt it be ſold to loſe 10 per cent.?

£ £ £
As 100 : 4 :: 90
4
100)360(3
300
60
20
100)1200(12 Ans. £3 12
1200

Or thus:
90
4
£3|60
20
s. 12|00

In caſting up the amount of goods bought, imported or exported; to the prime coſt of ſuch goods we muſt add all the charges upon them, in order to fix the price they ſtand us in.

3. Suppoſe I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to D360: I pay duty here 92c. per piece, for freight D12, and porterage D1 25c.: What does it ſtand me in per piece, and how muſt I ſell it per piece to gain D10 per cent.?

		D. c.
	Firſt coſt	360
12 bales.	Duty	110 40
10	Freight	12
	Porterage	1 25
120 pieces.	Whole coſt	483 65

Pieces. D. c. Piece. D.c.
As 120 : 483 65 :: 1 : 4 03 coſt per piece.

D. per ct. D.c. c. m.
Again, as 100 : 10 :: 4 03 : 40 3 gain.
40 3

D4 43 3 the price at which it muſt be ſold per piece.

4. Bought cloth, at D2 50c. per yard, which not proving ſo good as I expected, I am content to loſe 17½ per cent. by it: How muſt I ſell it per yard?

Ans. D2 6c. 2½m.

5. If 120lb. of ſteel coſt 7l. how muſt I ſell it per lb. to gain 15½l. per cent.?

As 120lb. : £7 :: 1lb. : 1s. 2d.
As £100 : 1s. 2d. :: £115½ : 1s. 4d. per lb. Ans.

6. Bought fiſh in Newburyport, at 10s. per quintal, and ſold it at Philadelphia, at 17s. 6d. per quintal; now allowing the charges at an average, or one with another, to be 2s. 6d. per quintal, and conſidering I muſt loſe 20l. per cent. by remitting my money home; what do I gain per cent.?

Selling price 17 6 Philadelphia currency, per quintal.
Charges 2 6 ditto.

15 0 ditto.
£. s. £ s.
As 100 : 15 :: 80 : 12 New England currency.
Sold at 12s. per quintal.
Bought at 10s per quintal.

Gained 2s. per quintal.
s. s. £. £.
As 10 : 2 :: 100 : 20 per cent. gained, Ans.

7. Bought 50 gallons of brandy, at 75c. per gallon, but by accident, 10 gallons leaked out: At what rate muſt I ſell the remainder per gallon, to gain upon the whole prime coſt, at the rate of 10 per cent.? Ans. D1 3c. $1\frac{1}{4}$m.

CASE III.

When there is gain or loſs per cent. to know what the commodity coſt.

Rule. As 100l. with the gain per cent. added, or loſs per cent. ſubtracted, is to the price; ſo is 100l. to the prime coſt.

Or, In *Federal Money*, divide the price with two cyphers annexed by D100 added to the gain, or leſs by the loſs, per cent. for the anſwer.

Examples.

1. If 1 yard of cloth be ſold, at D1 2c. and there is gained $13\frac{1}{3}$ per cent. what did the yard coſt?

D.c. D. c
As D100+$13\frac{1}{3}$: 1 2 :: 100 : 90 prime coſt, Anſwer.

Or, $\frac{102{\cdot}00}{113{\cdot}33\frac{1}{3}}=\cdot9$ Ans. as before.

2. If 12 yards of cloth are ſold at 15s. per yard, and there is 7l. 10s loſs per cent. in the ſale: What is the prime coſt of the whole?

Yd. s. yds. £
As 1 : 15 :: 12 : 9
£ s. £ £ £ s. d.
As 92 10 : 9 :: 100 : 9 14 7 Ans.

3. If 19½ cwt. ſugar be ſold at D14 50c. per cwt. and I gain D15 per cent.: What did it coſt per cwt.?

D. D.c. D. D.
As 115 : 14·50 :: 100 : 12·608 Ans.

CASE IV.

If by wares ſold at ſuch a rate, there is ſo much gained or loſt per cent. to know what would be gained or loſt per cent. if ſold at another rate.

Rule.

As the firſt price is to 100l. with the profit per cent. added, or loſs per cent. ſubtracted; ſo is the other price, to the gain or loſs per cent. at the other rate.

N. B. If your anſwer exceed 100, the exceſs is your gain per cent.; but if it be leſs than 100, the deficiency is your loſs per cent.

Examples.

1. If cloth ſold at D1 2c. per yard, be 13⅓ profit per cent. what gain or loſs per cent. ſhall I have, if I ſell the ſame at 90c. per yard?

D. c. D. c. D.
As 1 2 : 113⅓ :: 90 : 100
And 100—100=0 Ans. I neither gain nor loſe.

2. If cloth, ſold at 4s. per yard, be 10l. per cent. profit: What ſhall I gain or loſe per cent. if ſold at 3s. 6d. per yard?

s. £ s. d.
As 4 : 110 : 3 6
12 12
48 42 Then, 100l.—96¼l.=3¾l.
110
42)4620(96¼ Ans. I loſt 3¾l. per cent. by the laſt ſale.
432
300
288
12

3. If I ſell a gallon of wine for D1 50c. and thereby loſe 12 per cent.: What ſhall I gain or loſe per cent. if I ſell 4 gallons of the ſame wine for D6 75c.?

D. c. D. c. c.
As 6 : 88 :: 6 75 : 99. And 100—99=1 per cent. loſs.

EQUATION OF PAYMENTS

IS the finding a time to pay at once ſeveral debts due at different times, ſo that no loſs ſhall be ſuſtained by either party.

Rule I.*

Multiply each payment by the time at which it is due; then divide the ſum of the products by the ſum of the payments, and the quotient will be the equated time, or that required.

Examples.

1. A owes B D380 to be paid as follows, viz. D100 in 6 months, D120 in 7 months, and D160 in 10 months: What is the equated time for the payment of the whole debt?

100×6= 600
120×7= 840
160×10=1600

100+120+160=380)3040(8 months, Ans.
3040

2. A owes B 104l. 15s. to be paid in $4\frac{1}{2}$ months, 161l. to be paid in $3\frac{1}{2}$ months, and 152l. 5s. to be paid in 5 months: What is the equated time for the payment of the whole? Ans. 4 months and 8 days.

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* This rule is founded upon the ſuppoſition that the ſum of the intereſts of the ſeveral debts, which are payable before the equated time, from their terms to that time, ought to be equal to the ſum of the intereſts of the debts payable after the equated time, from that time to their terms.

3. There is owing to a merchant 998l. to be paid, 178l. ready money, 200l. at 3 months, and 320l. in 8 months; I demand the indifferent time for the payment of the whole? Ans. $4\frac{1}{2}$ months.

4. The ſum of D164 16c. 6m. is to be paid, $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months and $\frac{1}{6}$ in 12 months: What is the mean time for the payment of the whole?

Ans. $7\frac{2}{3}$ months.

Rule II.

See by Rule 1, at what time the firſt man, mentioned, ought to pay in his whole money; then as his money is to his time, ſo is the other's money, to his time, *inverſely*, which when found, muſt be added to, or ſubtracted from, the time at which the ſecond ought to have paid in his money, as the caſe may require, and the ſum, or remainder, will be the time of the ſecond's payment.

Examples.

1. A is indebted to B D150, to be paid, D50 at 4 months, and D100 at 8 months: B owes A D250, to be paid at 10 months: It is agreed between them that A ſhall make preſent pay of his whole debt, and that B ſhall pay his ſo much the ſooner, as to balance that favour: I demand the time at which B muſt pay the D250, reckoning ſimple intereſt.

$50 \times 4 = 200$
$100 \times 8 = 800$

$50+100=15|0)\,100|0(6\frac{2}{3}$ months, A's equated time.
90
10

D. mo. D. mo.
As 150 : $6\frac{2}{3}$ :: 290 : 4
mo. mo. mo.

Then, $10-4=6$ time of B's payment.

2. A merchant has D1200 due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the reſt at 6 months; but the debtor agrees to pay $\frac{1}{2}$ down: How long may the debtor detain the other half, ſo that neither party may ſuſtain loſs?

mo. mo.
$\frac{1}{6}\times2=0\frac{1}{3}$
$\frac{1}{3}\times3=1$
$\frac{1}{2}\times6=3$

Equated time=$4\frac{1}{3}$.

Now, as $\frac{1}{2}$ was paid $4\frac{1}{3}$ months before it was due, it is reaſonable that he ſhould detain the other $\frac{1}{2}$ $4\frac{1}{3}$ months after it became due, which, added, gives $8\frac{2}{3}$ months, the true time for the ſecond payment.

POLICIES OF INSURANCE.

INSURANCE is a ſecurity, or aſſurance, by mean of a writ called a *Policy*, to indemnify the inſured of ſuch loſſes as ſhall be ſpecified in the policy ſubſcribed by the inſurer, or inſurers, by which the under writers oblige themſelves to make good and effectual the property inſured, in conſideration of a certain premium at a ſtipulated rate per cent. (which varies according to the riſk) to be immediately paid down, or otherwiſe ſecured according to the tenor of the agreement.

The average loſs is 10 per cent.; that is, if the inſured ſuffer any damage or loſs, not exceeding 10 per cent. he bears it himſelf, and the inſurers are free.

A policy ſhould be taken out for a ſum ſufficient to cover the principal and premium, and the buſineſs of this rule is, in general, to calculate what that ſum ſhould be.

CASE I.

When the premium, at a certain rate per cent. for inſuring a ſum is required, the operation is the ſame as in intereſt, or commiſſion.

EXAMPLES.

1. What is the premium upon 537l. 15s. 9d. at $6\frac{1}{2}$ per cent.?

£ s. d.
537 15 9
6½

3226 14 6
½= 268 17 10½

34|95 12 4½
20

19|12
12

1|48
4

1|94 Ans. 34l. 19s. 1½d. nearly.

2. What is the premium upon D375, at 7½ per cent. ?

D375
·075

1875
2625

Ans. D28·125

CASE II.

To find the ſum for which a policy ſhould be taken out to cover a given ſum.

Rule.

Take the premium from 100l. (or in *federal money* D100) and ſay, As the remainder is to 100, ſo is the ſum adventured to the policy.* Or,

---

* Now it is plain, that if I want to recover 92l. I muſt in this caſe, inſure upon 100l.: therefore to recover 759l. I muſt inſure upon 825l.; for when 8 per cent. for premium is deducted, I ſhall have 759l. remaining nett.

For £825=ſum inſured upon at 8 per cent.
66=premium to be deducted.

759=the firſt outſet.

In *decimals*, take the premium from 100, annex two cyphers to the ſum to be covered, and divide by the remainder for the policy.

EXAMPLES.

1. It is required to cover 759l. premium 8 per cent.: For what ſum muſt the policy be taken?

100
8
———
92 : 100 :: 759
100
———
92)75900(825l. Ans.
736
———
230
184
———
460
460

Or, $\frac{75900}{92}$ =825l. Ans. as before.

2. A merchant ſent a veſſel and cargo to ſea, valued at D5760: What ſum muſt the policy be taken out for, to cover this property, premium $19\frac{1}{2}$ per cent.?

100
19·5
———
D. D. D. c.
80·5 : 100 :: 5760 : 7155·28—Ans.

Or, $\frac{576000}{80{\cdot}5}$ =D7155·28c.— Ans. as before.

CASE III.

When a policy is taken out for a certain ſum, in order to cover a given ſum.

RULE.

To find the premium, ſay, As the policy is to the covered ſum; ſo is £100 (or D100) to a fourth number, which, being taken from 100, will leave the premium.

Or, In *decimals*, divide the ſum covered, with two cyphers annexed, by the policy; ſubtract the quotient from 100, and the remainder is the premium.

EXAMPLES.

1. If a policy be taken out for 1250l. to cover 500l. what is the premium per cent.?

1250 : 500 :: 100
100

1250)50000(40, and 100l.—40=60l. Ans.

Or, $\frac{50000}{1250}$=40, &c as before.

2. If a policy be taken out for D781·25, to cover D625: Required the premium per cent.

D.	D.	D.	D. c.
As 781·25 :	625 ::	100 :	87 50

And 100—87·5=12 5, or 12½ per cent. premium, Ans.

Or, $\frac{62500}{781 \cdot 25}$=87·5, &c. as before.

CASE IV.

When the policy for covering any ſum and the premium per cent. are given, to find the ſum to be covered.

RULE.

Deduct the premium per cent. from 100, and ſay, As 100 is to the remainder, ſo is the policy to the ſum required to be covered. Or,

In *decimals*, multiply the policy by the remainder found as before, and point off two right hand places in the product for the anſwer.

EXAMPLES.

1. If a policy be taken out for £1250 at 60 per cent. what is the adventure, or ſum to be covered?

100
60

100 : 40 :: 1250
40

100)50000(500l. Ans.

Or, $1250\times\overline{100-60}=50000$, and, pointing off two places, 500·00 Anſwer, as before.

2. If a policy be taken out for D781 25c. at $12\frac{1}{8}$ per cent. required the ſum covered?

$$\text{As } 100 : \overline{100-12\tfrac{1}{8}} :: 781{\cdot}25 : \frac{781{\cdot}25\times\overline{100-12\tfrac{1}{8}}}{100} =$$

D625 Anſwer.

Or, $781{\cdot}25\times\overline{100-12\,5}=62500$; and 625·00 Ans. as before.

COMPOUND INTEREST

IS that which ariſes from the intereſt being added to the principal, and (continuing in the hands of the borrower) becoming part of the pricipal, at the end of each ſtated time of payment.

METHOD I.

RULE.*

Find the *amount* of the given principal, for the time of the firſt payment, by Simple Intereſt : next, find the intereſt of that ſum, or principal, and add it as before, and thus proceed for any number of years, ſtill accounting the laſt amount as the principal for the next payment. The given principal being ſubtracted from the laſt amount, the remainder will be the compound intereſt.

In *federal money*, multiply the principal by the rate for the firſt time of payment, ſetting the product two places more to the right than the multiplicand, and the decimal point in the product under that in the multiplicand ; then find the amount, and proceed as above.

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* It may be obſerved that all computations, relating to Compound Intereſt, are founded upon a ſeries of terms, increaſing in Geometrical Progreſſion, wherein the number of years aſſigns the index of the laſt and higheſt term : Therefore, as one pound is to the amount of one pound, for any given time, ſo is any propoſed principal, or ſum, to its amount for the ſame time.

NOTE. It is not uſually neceſſary to carry the work beyond mills; therefore, when the figure next beyond mills, at the right, exceeds 5, increaſe the number of mills 1; when it does not exceed 5 it may be omitted. The reſult will be exact enough for common purpoſes.

EXAMPLES.

1. What will $\left\{\begin{matrix}\text{£480}\\ \text{D1600}\end{matrix}\right\}$ amount to in 5 years, at 6 per cent. per annum?

Principal	£480
Rate of intereſt	6
	28|80
	20
	16|00

Principal for the firſt year	£480	0
Intereſt of ditto	28	16
Principal for the ſecond year	508	16
		6
	30|52	16
	20	
	10|56	
	12	
	6|72	
	4	
	2|88	

Principal for the ſecond year	£508	16	0
Intereſt for ditto	30	10	6½
Principal for the third year	539	6	6½
			6
	32|35	19	3
	20		
	7|19		
	12		
	2|31		
	4		
	1|24		

Principal for the 3d year	£539	6	6½
Intereſt for ditto	32	7	2¼
Principal for the 4th year	571	13	8¾
			6
	34|30	2	4½
	20		
	6|02		
	12		
	0|28		
	4		
	1|14		

Principal for the 4th year	£571	13	8¾
Intereſt for do.	34	6	0¼
Principal for the 5th year	605	19	9
			6
	36|35	18	6
	20		
	7|18		
	12		
	2|22		

Principal for the 5th year	£605	19	9
Intereſt for ditto	36	7	2
Amount for 5 years	642	6	11
Subtract the firſt principal	480	0	0
Compound intereſt for 5 years	162	6	11

In Federal Money thus:

Principal for the firſt year	D1600
Rate of intereſt	6
Intereſt firſt year	96·00
Amount 1ſt and principal 2d year	1696·

Carried over.

Brought over	1696·
	6
Intereſt 2d year	101·76
Amount 2d year, principal 3d	1797·76
	6
Intereſt 3d year	107·8656
Amount 3d year, principal 4th	1905·6256
	6
Intereſt 4th year	114·337536
Amount 4th year, principal 5th	2019·963136
	6
Intereſt 5th year	121·19778816
Amount for 5 years	2141·16092416
Subtract firſt principal	1600·
Compound intereſt for 5 years =	541·16092416

Or thus:

	D.
Firſt principal	1600·
	6
Intereſt	96·00
Second principal	1696·
	6
Intereſt	101·76
Third principal	1797·76
	6
Intereſt	107·866
Fourth principal	1905·626

Fourth principal	D1905·626
	6
Intereſt	114·338
Fifth principal	2019·964
	6
Intereſt	121·198
Amount	2141·162
1ſt principal	1600·
Compound intereſt	541·162 nearly, as before.

2. What is the compound intereſt of D740 for 6 years, at 4 per cent. per annum? Ans. D196 33c. 1m.

3. What will $\left\{\begin{array}{l}\text{£150}\\ \text{D500}\end{array}\right\}$ amount to in a year, at 2 per cent. a month? Ans. $\left\{\begin{array}{l}\text{£190 4s. 5d.}\\ \text{D634 12c. 6m.}\end{array}\right.$

METHOD II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the firſt year, and the principal for the ſecond.

2. In like manner find the amount for every ſucceeding year.

When the rate is at 6 per cent. per annum.

1. Divide the principal by 20, and that quotient by 5: theſe quotients, added to the principal, will be the amount for the firſt year, and the principal for the ſecond.

2. In like manner obtain the amount for every ſucceeding year.

EXAMPLES.

1. What is the amount of 480l. at 6 per cent. per annum, for 5 years?

20)480
5) 24
4 16

20)508 16 amo. of 1ſt year.
5) 25 8 9½
5 1 9

20)539 6 6½ ditto of 2d.
5) 26 19 3¾
5 7 10¼

20)571 13 8½ ditto of 3d.
5) 28 11 8¼
5 14 4

20)605 19 8¼ ditto of 4th.
5) 30 5 11¾
6 1 2¼

£642 6 10¼ do. 5th, Ans.

2. Of the ſame ſum at 5 per cent. for 5 years?

20)480
24

20)504 amount of 1ſt yr.
25 4

20)529 4 ditto of 2d.
27 9 2¼

20)555 13 2¼ ditto of 3d.
26 15 7¾

20)583 8 10 do. of 4th.
29 3 5¼

£612 12 3¼ do. of 5th, Ans.

NOTE. The ſame may be done in Federal Money, but the firſt method is generally more eaſy.

COMPOUND INTEREST BY DECIMALS.

A TABLE of the amount of £1 or D1, at ½ per cent. per month, as practiſed at the Banks.

Mon.	£ or D. dec pts.	Mon.	£ or D. dec. pts.	Mon.	£ or D. dec. pts.
1	1·005	5	1·025	9	1·045
2	1·01	6	1·03	10	1·05
3	1·015	7	1·035	11	1·055
4	1·02	8	1·04	12	1·06

A TABLE of the Amount of £1 or D1, from 1 Day to 31 Days, at 6 per cent. per annum.

Days.	£ or D. dec. pts.	Days	£ or D. dec. pts.	Days	£ or D. dec. pts.
1	1·00016	12	1·00197	22	1·00361
2	1·00032	13	1·00213	23	1·00378
3	1·00049	14	1·0023	24	1·00394
4	1·00065	15	1·00246	25	1·0041
5	1·00082	16	1·00263	26	1·00427
6	1·00098	17	1·00279	27	1·00443
7	1·00115	18	1·00295	28	1·0046
8	1·00131	19	1·00312	29	1·00476
9	1·00147	20	1·00328	30	1·00493
10	1·00164	21	1·00345	31	1·00509
11	1·00180				

CASE I.

When the principal, the rate of intereſt, and time, are given, to find either the amount or intereſt.

Rule.

1. Find the amount of £1 or D1 for one year at the given rate per cent.

2. Involve the amount, thus found, to ſuch power, as is denoted by the number of years; or, in Table I, at the end of Annuities, under the rate and againſt the given number of years, you will find the power.

3. Multiply this power by the principal, or given ſum, and the product will be the amount required, from which, if you ſubtract the principal, the remainder will be the intereſt.

Examples.

1. What is the Compound intereſt of 600l. for 4 years, at 6 per cent. per annum?

1·06= { amount of 1l. for 1 year, at 6 per cent. per ann.
Multiply by 1·06

1·1236=2d power.
Multiply by 1·1236

1·26247696=4th power.
Multiply by 600=principal.

757·48617600=amount.
Subtract 600

157·486176=£157 9s. 8¼d. intereſt required.

By TABLE I.

Tabular amount of 1l. for 4 years, at 6 per cent. per annum, } =1·2624769
Multiply by the principal= 600

Amount=757·4861400

2. What is the amount of D1500 for 12 years at 3½ per cent. per annum?

D1·035=amount of D1 for 1 year at 3½ per cent. per annum.

And, $1·035^{12}$ × 1500=D2266 60c. nearly, Ans.

ANOTHER METHOD

Of working compound intereſt for *years*, *months*, and *days*, which is much more conciſe than the preceding method.

RULE.

To the logarithm of the principal, found in any Table of logarithms, add the ſeveral logarithms, anſwering to the number of *years*, *months*, and *days*, found in the following tables, and their ſum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number correſponding thereto will be the anſwer.

LOGARITHMICK TABLES, at 6 per cent. per annum, for Years, Months, and Days.

Years.	dec. pts	Y.	dec pts.	Months.	dec pts.
1	·025306	21	·531426	1	·002166
2	·050612	22	·556732	2	·004321
3	·075918	23	·582038	3	·006466
4	·101224	24	607344	4	·0086
5	·12653	25	·63265	5	·010724
6	·151836	26	·657956	6	·012837
7	·177142	27	·683262	7	·01494
8	202448	28	·708568	8	·017033
9	·227754	29	·733974	9	·019110
10	·25306	30	·75918	10	·021189
11	·278366	31	·784586	11	·023252
12	·303672	32	·809792		
13	·328978	33	·835098		
14	354284	34	860404		
15	·37959	35	·88571		
16	·404896	36	911016		
17	·430202	37	936322		
18	·455508	38	·961628		
19	·480814	39	·986934		
20	·50612	40	1·01224		
Days.		D.		Days	
1	·000071	12	000857	22	·001568
2	·000143	13	·000928	23	·001639
3	·000215	14	·000999	24	·00171
4	·000287	15	·00107	25	·001781
5	·000358	16	·001142	26	·001852
6	·000429	17	·001213	27	001923
7	·0005	18	·001284	28	·001994
8	·000571	19	·001355	29	·002065
9	000642	20	·001426	30	·002136
10	·000713	21	·001497	31	·002207
11	·000785				

What is the amount of £132 10s. at 6 per cent. per annum, for 9 years, 8 months and 15 days?

To the log. of £132·5=2·122216

$$\text{Add}\begin{cases}\text{Log. for 9 years} & = \cdot227754\\ \text{ditto for 8 months} & = \cdot017033\\ \text{ditto for 15 days} & = \cdot00107\end{cases}$$

2·368073

$$\left.\begin{array}{l}\text{Becauſe 8 months are paſt, deduct 4}\\ \text{per cent. upon the log. of 15 days}\end{array}\right\}=\cdot0000428$$

Remains 2·3680302

The neareſt to which, in the table of logarithms, is 2·368101, and the natural number anſwering thereto is 233·4=£233 8s. Ans.

CASE II.

When the amount, rate and time, are given, to find the principal.

RULE.

Divide the amount by the amount of £1 or D1 for the given time, and the quotient will be the principal.

Or, If you multiply the preſent value of £1 or D1 for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or preſent worth.

EXAMPLES.

1. What is the preſent worth of 757l. 9s. 8½d. due 4 years hence, diſcounting at the rate of 6l. per cent. per annum?

BY TABLE I.

$$\left.\begin{array}{l}\text{Divide by the tabular}\\ \text{am. of 1l. for 4 years}\end{array}\right\}=1\cdot2624769)757\cdot4861400(\text{£}600$$

Ans.

BY TABLE II.

Amount=757·48614

$$\left.\begin{array}{l}\text{Mult. by the preſent worth of 1l.}\\ \text{for 4 years, at 6 per cent. per an.}\end{array}\right\}=\cdot7920936$$

Ans. 599·999923582704+=£600.

2. What principal muſt be put to intereſt 6 years, to amount to D689·4214033809453125, at 5½ per cent. per annum? Ans. D500.

CASE III.

When the principal, rate and amount, are given, to find the time.

Rule.

Divide the amount by the principal; then divide this quotient by the amount of £1 or D1 for 1 year, this quotient by the ſame, till nothing remain, and the number of the diviſions will ſhow the time.

Or, Divide the amount by the principal, and the quotient will be the amount of £1 or D1 for the given time, which ſeek under the given rate in Table 1, and, in a line with it, you will ſee the time.

Example.

In what time will D500 amount to D689 42c. 1m.+, at $5\frac{1}{2}$ per cent. per annum?

	500	689·421+
	1·055	1·379—
	1·055	1·307—
	1·055	1·239—
6 diviſions.	1·055	1·174+
	1·055	1·113—
	1·055	1·053+
		1·—

Ans. 6 years.

CASE IV.

When the principal, amount and time, are given, to find the rate per cent.

Rule.

Divide the amount by the principal, and the quotient will be the amount of 1l. or D1 for the given time; then, extract ſuch root as the time denotes, and that root will be the amount of 1l. or D. for one year, from which ſubtract unity, and the remainder will be the ratio.

Or, Having found the amount of 1l. or D. for the time as above directed, look for it in Table 1ſt, even with the given time, and directly over the amount you will find the ratio.

EXAMPLE.

At what rate per cent per annum will D500 amount to D689·421403+ in 6 years?

$$\frac{689\cdot42\,403+}{500}=1\cdot378843-\text{; and }\sqrt[6]{1\cdot378843-}=1\cdot055.$$

Then $1\cdot055-1=\cdot055=$ ratio. Hence the rate is 5½ per cent. per annum, Anſwer.

DISCOUNT BY COMPOUND INTEREST.

The ſum, or debt to be diſcounted, the time and rate, given, to find the preſent worth.

RULE.

Divide the debt by that power of the amount of 1l. or D for 1 year, denoted by the time, and the quotient will be the preſent worth, which ſubtracted from the debt, will leave the diſcount.

EXAMPLES.

1. What is the preſent worth, and diſcount, of 600l. due 3 years hence at 6l. per cent. per annum, compound intereſt?

Divide by $\overline{1\cdot06}|^{3}=1\cdot19101)600\cdot00000(503\cdot7741=$ £503 15s. 5¼d preſent worth, and £600—£503 15s. 5¼d.= £96 4s. 6¼d. = diſcount.

$$\text{Or, }\frac{600}{1\cdot19101}=£503\cdot7741\text{, and }600-\frac{600}{1\cdot19101}=£96\cdot2259.$$

By TABLE II.

In this Table, correſponding to the time and rate, we have

·839619= } preſent worth of 1l. for the time and rate.

Multiply by 600=debt, or principal.

503·771400=preſent worth of the debt.

2. What ready money will diſcharge a debt of D1000 due 4 years hence, at 5 per cent. per annum, compound intereſt? Ans. D822·7024+

ANNUITIES OR PENSIONS, IN ARREARS, AT COMPOUND INTEREST.

CASE I.

When the annuity, or penſion, the time it continues, and the rate per cent. are given, to find the amount.

Rule.

1. Make 1 the firſt term of a Geometrical Progreſſion, and the amount of 1l. or D. for 1 year at the given rate per cent. the ratio.

2. Carry the ſeries to ſo many terms as the number of years, and find its ſum.

3. Multiply the ſum thus found by the given annuity, and the product will be the amount ſought.

Or, multiply the amount of £1 or D1 for 1 year into itſelf ſo many times as there are years leſs by 1; then multiply this product by the annuity; and ſubtract the annuity therefrom. Laſtly, divide the remainder by the ratio leſs 1, and the quotient will be the amount.

Examples.

1. What will an annuity of 60l. per annum, payable yearly, amount to in 4 years, at 6l. per cent.?

Firſt Method.

$1+1{\cdot}06+\overline{1{\cdot}06}|^{2}+\overline{1{\cdot}06}|^{3}=4{\cdot}374616$=ſum.

Multiply by 60=annuity.

262·476960
20

9·53920
12

6·4704
4

1·8816 Ans. £262. 9s 6¼d.

Or, $1+1{\cdot}06+\overline{1{\cdot}0\text{[illegible]}}^{2}+\overline{1{\cdot}06}|^{3}\times 60$=£262 9s. 6¼d.

Second Method.

1·06×1·06×1·06×1·06=1·26247
Multiply by 60 annuity.

75·74820
Subtract 60

Divide by 1·06—1=·06)15·7482(262·47=£262 9s. 4¾d.
12

37
36

14
12

28
24

42
42

$$\text{Or, } \frac{\overline{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times60}-60}{1{\cdot}06-1} = £262{\cdot}47$$

Or, by TABLE. III.

Multiply the tabular number under the rate and oppoſite to the time by the annuity, and the product will be the amount.

2. What will an annuity of 60l. per annum amount to in 20 years, allowing 6l. per cent. compound intereſt?

Under 6l. per cent. and oppoſite 20, in Table. 3, you will find

Tabular number=36·78559
Multiply by 60=annuity.

2207·13540=£2207 2s. 8¼d. Ans.

3. What will wages, of D25 per month, amount to in a year, at ½ per cent. per month?

Ans. D308 38c. 9m.

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, penſion, &c.

Rule.

Multiply the whole amount by the amount of £1 or D1, for a year, from which ſubtract the whole amount, divide the remainder by that power of the amount of £1 or D1 for a year, ſignified by the number of years, made leſs by unity, and the quotient will be the anſwer.

Or, Find the amount of an annuity of £1 or D1, for the given time and rate (by Caſe I.); divide the given ſum by this amount; and the quotient will be the annuity required.

Examples.

1. What annuity, being forborne 4 years, will amount to £262·47696, at 6 per cent. compound intereſt?

1·06
1·06
———
636
1060
———
1·1236
1·06
———
67416
112360
———
1·191016
1·06
———
7146096
11910160
———
1·26247696
Subtract 1·
———
·26247696=Diviſor.

262·47696=amount
Mult. by 1·06=am. of 1l. for 1 yr.
———
157486176
262476960
———
278·2255776
Subt. 262·47696
———
·26247696)15·7486176(60l. Ans.
15·7486176
———
0

Or, $\dfrac{262{\cdot}47696\times1{\cdot}06-262{\cdot}47696}{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06-1}=60.$

Or thus.

Amount of an annuity of 1l. for 4 years, at 6 per cent. per annum,=4·374616 (by Caſe I); and

$$\frac{262\cdot47696}{4\cdot374616}=\pounds60\ \text{Ans.}$$

Or, by Table III. the amount of 1l. is found to be 4·374616; and the anſwer is found as before.

2. What annuity, being forborne 20 years, will amount to D2207·1354, at 6 per cent. compound intereſt?

Amount of an annuity of D1 for 20 years at 6 per cent. per annum=36·78559: And

36·78559)2207·13540(D60, Ans.
2207·1354

0

CASE III.

When the annuity, amount and ratio are given, to find the time.

RULE.

Multiply the amount by the ratio, to this product add the annuity, and from the ſum ſubtract the amount; this remainder being divided by the annuity, the *quotient* will be that power of the ratio ſignified by the time, which being divided by the amount of 1l. for one year, and this *quotient* by the ſame, till nothing remain, the number of thoſe diviſions will be equal to the time. Or, look for this number under the given rate in Table 1, and in a line with it, you will ſee the time. Or,

Divide the amount by the annuity; from the quotient ſubtract 1: from the remainder ſubtract the ratio; from ſucceſſive remainders, ſubtract the ſquare, cube, &c. of the ratio, till nothing remain; and the whole number of the ſubtractions will be the anſwer. Or, find the quotient in Table III. under the rate, and in a line with it ſtands the anſwer.

EXAMPLES.

1. In what time will 60l. per annum, payable yearly, amount to £262·47696, allowing 6l. per cent. compound intereſt for the forbearance of payment?

262·47696=amount.
Multiply by 1·06=ratio.

157486176
262476960

278·2255776
Add 60· =annuity.

338·2255776
Subtract 262·47696

Divide by 60)75·7486176

Divide by 1·06)1·26247696

Divide by 1·06)1·191016

Divide by 1·06)1·1236

Divide by 1·06)1·06

1 Ans. 4 years.

The number of diviſions by 1·06, being 4, gives the number of years=4, the anſwer.

Or thus :
Annuity=60)262·47696=amount.

4·374616
1. Subtract 1·

3·374616
2. Subtract 1·06 =ratio.

2·314616
3. Subtract 1·1236 $= \overline{\text{ratio.}}|^{2}$

1·191016
4. Subtract 1·191016 $= \overline{\text{ratio.}}|^{3}$

· · · · · Anſwer 4 years.

Or, looking into Table III. under the rate, 6, the quotient, 4·374616, ſtands againſt 4 years, anſwer, as before.

Or, in Table I. under the given rate, you will find 1·262476, and in a line, under years, you will find 4.

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given, to find the preſent worth.

RULE *

1. Divide the annuity by the ratio, or the amount of D1 or £1 for 1 year, and the quotient will be the preſent worth of 1 year's annuity.

2 Divide the annuity by the ſquare of the ratio, and the quotient will be the preſent worth for 2 years.

3. In like manner, find the preſent worth of each year by itſelf, and the ſum of all theſe will be the preſent value of the annuity, ſought.

Or, divide the annuity, &c. by that power of the ratio ſignified by the number of years, and ſubtract the quotient from the annuity; this remainder being divided by the ratio leſs 1, the quotient will be the preſent worth.

EXAMPLES.

1.† What ready money will purchaſe an annuity of 60l. to continue 4 years, at 6l. per cent. Compound intereſt?

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[*] The *amount* of an annuity may alſo be found for years and parts a of year, thus:

1. Find the amount for the whole years, as before.

2. Find the intereſt of that amount for the given parts of a year.

3. Add this intereſt to the former amount, and it will give the whole amount required.

The *preſent worth* of an annuity for years and parts of a year may be found thus:

1 Find the preſent worth for the whole years, as before.

2. Find the preſent worth of this preſent worth, diſcounting for the given parts of a year, and it will be the whole preſent worth required.

[†] Queſtions in this caſe may alſo be anſwered by firſt finding the amount of the given annuity by Caſe I. of annuities in arrears, p. 240, and then the preſent worth, or principal, by Caſe II. of Compound Intereſt, page 246.

Firſt Method.

Ratio = 1·06)60·00000(56·603=preſent worth for 1 year.

$\overline{\text{Ratio}}|^{2}$ = 1·1236)60·00000(53·399= do. for 2 years.

$\overline{\text{Ratio}}|^{3}$ = 1·191016)60·00000(50·377= do. for 3 years.

$\overline{\text{Ratio}}|^{4}$ =1·26247696)60·00000(47·525= do. for 4 years.

Ans. 207·904=£207 18s. 0¾d.

Second Method.

4th power of the ratio } =1·26247696)60·0000000(47·525

From 60

Subtract 47·525

Diviſor=1·06—1=·06)12·475

207·916=£207 18s. 3¾d.

Or, $\frac{60}{\overline{\cdot06}|^{4}}$ =47·525   60—47·525=12·475

And $\frac{12\cdot475}{\cdot06}$=207·916.

By TABLE III.

Under 6l. per cent. and oppoſite 4 we find

4·37461=amount of 1l. annuity for 4 years.

Multiply by 60=annuity.

262·47660=amount of £60 for 4 years.

Then oppoſite 4 years and under £6 per cent. in Table 2,

We have ·792093

Multiply by 262 7466

4752558

4752558

3168372

5544651

1584186

4752558

1584186

208·1197426338=£208 2s. 4½d.

Or, oppoſite 4 years, and under £6 per cent. in Table 1ſt, we have 1·26247=the amount of £1 for 4 years

Then, 262·7466÷1·26247=208·1209=£208 2s. 5d.

By Table IV.

Multiply the tabular number, under the rate, and oppoſite the time, into the annuity, and the product will be the preſent worth.

Thus, in Example 1ſt. What ready money will purchaſe 60l. annuity, to continue 4 years, at 6l. per cent. compound intereſt?

Under 6l. per cent. and even with 4 years,

We have 3·4651=preſent worth of 1l. for 4 years.
Multiply by 60=annuity.

Ans.=207·9060=£207 18s. 1¼d.

2. What is the preſent worth of an annuity of D60 per annum, to continue 20 years, at 6 per cent. compound intereſt? Ans. D688·65 (nearly.)

CASE II.

When the preſent worth, time and rate are given, to find the annuity, rent, &c.

Rule.

1. From that power of the ratio, denoted by the number of years, plus 1, ſubtract that power of it denoted by the number of years.

2. Divide the remainder by that power of the ratio, ſignified by the time made leſs by unity.

3. Multiply the preſent worth into this quotient, and the product will be the annuity, penſion, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the preſent worth.

2. Multiply that power of the ratio, denoted by the time, by the preſent worth, and ſubtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made leſs by unity, and the quotient will be the annuity.

Examples.

1. What annuity, to continue 4 years, will £207·904 purchaſe, compound intereſt, 6l. per cent.?

Firſt Method.

From 1·06×1·06×1·06×1·06×1·06=1·3382255776
Subt. 1·06×1·06×1·06×1·06 =1·26247696

Divide by $\overline{1\cdot06}|^{4}$ —1=·26247696)·0757486176(·2885898
·2885898
Multiply by 207·9 preſent worth.

25973082
20201286
57717960

Ans. 59·99781942=£60

Second Method.

From 1·06×1·06×1·06×1·06×1·06×207·9=278·217097573
Take 1·06×1·06×1·06×1·06×207·9 =262·268959984

Divide by $\overline{1\cdot06}|^{4}$ —1= 26247696)15·748157589(
Quotient=59·998=£60.

By Table V.

Multiply the tabular number, correſponding with the rate and time, by the purchaſe money, and the product will be the annuity

Under 6l per cent and oppoſite 4 years, you will find
·28859=annuity which 1l. will purchaſe in 4 years.
Multiply by 297·9

259731
202013
577180

59 997861=£60 Ans.

2. What ſalary to continue 20 years, will D688 95c. purchaſe, at 6 per cent. compound intereſt? Ans. D60.

CASE III.

When the annuity, preſent worth and ratio, are given, to find the time.

Rule.

Divide the annuity by the product of the preſent worth and ratio ſubtracted from the ſum of the preſent worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which, being divided by the ratio, and this quotient by the ſame, till nothing remain, the number of diviſions will ſhow the time: Or, the above quotient being ſought in Table 1ſt. under the given rate, in a line with it, you will ſee the time.

EXAMPLES.

1. For how long may an annuity of 60l. per annum be purchaſed for £207·906336762, at 6 per cent. compound intereſt?

Multiply 207·906336762
by 1·06

12474380205 72
2079063367620

220·38071696772

To 207·906336762=preſent worth
Add 60· =annuity.

From 267·906336762
Take 220·380716967

47·525619795=diviſor.

47·525619795)60·000000000(1·26247696

Divide by 1·06)1·26247696

1·06)1·191016

1·06)1·1236

1·06)1·06

1 $\left\{\text{The number of diviſions} = \text{time} = 4 \text{ years.}\right.$

Or, $\dfrac{60}{\overline{207\cdot906336762+60}-\overline{207\cdot906336762\times1\cdot06}}=$

1·26247696, which being ſought in Table I. under the given rate, in a line with it, is 4=4 years.

2, How long may a leaſe of D300 yearly rent, be had for D2132·341, allowing 5 per cent. compound intereſt, to the purchaſer ? Ans. 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

When the annuity, time and ratio, are given, to find the preſent worth of the annuity in reverſion.

RULE.

1. Divide the annuity by that power of the ratio denoted by the time of its continuance.

2. Subtract this quotient from the annuity : divide by the ratio leſs 1, and the quotient will the preſent worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reverſion, (or, time to come, before the annuity commences) and the quotient will be the preſent worth of the annuity in reverſion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reverſion, and the ratio leſs 1, continually together for a diviſor, and the quotient ariſing from the diviſion of theſe two numbers will be the preſent worth of the annuity in reverſion.

EXAMPLES.

1. What is the preſent worth of 60l. payable yearly, for 4 years ; but not to commence till two years hence, at 6l. per cent. ?

Firſt Method.

Ratio=1·06
1·06
———
636
1060
———
2d power=1·1236 Carried over.

2d power=1·1236
1·1236

67416
33708
22472
11236
11236

Divide by 4th power=1·26247696)60·000000000000(
Subtract the quotient=47·525619794281

Divide by 1·06—1=·06)12·474380205719

Divide by 1·06×1·06=1·1236)207·9063367619(

Quotient=185·035899=
£185 0s. 8½d.=preſent worth of the annuity in reverſion.

Or, in Table IV. find the preſent value of 1l. at the given rate, both for the time in being and the time in reverſion added together, and ſubtract the preſent worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the anſwer.

Preſent worth of the time in being and reverſion=4·91732
Preſent worth of the time in being=1·8333

3·08402
60

Ans. £185·04120

$$\text{Or, } \frac{60}{1{\cdot}26247696}=47{\cdot}5256 \qquad \frac{60-47{\cdot}5256}{1{\cdot}06-1}=207{\cdot}906$$

$$\text{And } \frac{207{\cdot}906}{1{\cdot}1236}=185{\cdot}035899$$

Second Method.

·26247696=4th power—1
Multiply by 60=annuity.

15·74861760=dividend.
1·26247696=4th power.
1·1236=2d power.

757486176
378743088
252495392
126247696
126247696

1·418519112256
·06=ratio—1

·08511114673536=diviſor.
·08511115)15·74861760(185·036 Ans.

$$\text{Or, } \frac{\overline{1\cdot06}|^4-1\times60}{\overline{1\cdot06}|^4\times\overline{1\cdot06}|^2\times\overline{1\cdot06-1}}=185\cdot036$$

2. What is the preſent worth of a reverſion of a leaſe of D60 per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6 per cent. to the purchaſer? Ans. D431·782 (nearly.)

An annuity, ſeveral times in reverſion, and rate being given, to find the ſeveral preſent values.

Find the preſent value of £1 or D1 by Table 4, at the given rate, and for the ſeveral given times, which, being ſeverally multiplied, by the annuity, the products will be the ſeveral preſent values of that annuity, for the ſeveral times given: ſubtract the ſeveral preſent values, the one from the other, and the ſeveral remainders will anſwer the queſtion.

3. A has a term of 6 years in an eſtate at 60l. per annum B has a term of 14 years in the ſame eſtate, in reverſion, after the 6 years are expired : and C has a further term of 16 years, after the expiration of 20 years. I demand the preſent values of the ſeveral terms, at 6 per cent. ?

£ s. d.
Preſ. value of £1 for 36 yrs=14·61722×60=877 0 $7\frac{8}{4}$
Ditto of ditto for 20 years =11·46992×60=688 3 $10\frac{3}{4}$
Ditto of ditto for 6 years = 4·91732×60=295 0 $9\frac{1}{4}$= A's term Therefore, 877 0 $7\frac{3}{4}$—688 3 $10\frac{3}{4}$=£188 16s. 9d. C's term, and 688 3 $10\frac{3}{4}$—295 0 $9\frac{1}{4}$=£393 3 $1\frac{1}{2}$=B's term.

4. For a leaſe of certain profits for 7 years, A offers to pay D300 gratuity, and D300 per annum, B offers D800 gratuity and D250 per annum, C bids D1300 gratuity and D200 per annum, and D bids D2500 for the whole purchaſe, without any yearly rent ; which is the beſt offer, computing at 6 per cent. ? D.

By Table IV. the preſent worth of D300 per annum, for 7 years, at 6 per cent. is } 1674·714
To which add 300·

Value of A's offer=1974·714

Preſent worth of D250 per annum for 7 years=1395·595
To which add 800·

Value of B's offer=2195·595

Preſent worth of D200 per annum for 7 years=1116·476
To which add 1300·

Value of C's offer=2416·476

D's offer=2500·

Hence it appears that D's offer is the beſt.

The above queſtions may be anſwered by the 4th and 2d Tables.

Take queſtion 1ſt for example.

1. Multiply the tabular number in Table 4, correſponding to the rate and the time of continuance, into the annuity, and the product will be the preſent worth, to commence immediately.

2. Multiply this preſent worth by the tabular number in Table 2, correſponding to the rate and the time of reverſion, and the product will be the preſent worth of the annuity in reverſion.

In Table 4th we have 3·4651
Multiply by 60=annuity.

207·9060
In Table 2 we have ·889996

1247436
1871154
1871154
1871154
1663248
1663248

185·035508376=preſent worth of the reverſion.

TABLE FIRST.

Showing the amount of 1l. or D.1 from 1 year to forty years.

ys	5 per cent	6 per cent	ys	5 per cent.	6 per cent.
1	1·0500000	1·0600000	21	2·7859625	3·3995636
2	1·1025000	1·1236000	22	2·925260	3·6035374
3	1·1576250	1·1910160	23	3·0715237	3·8197496
4	1·215506	1·2624769	24	3·2250999	4·0489346
5	1·2762815	1·3382256	25	3·3863549	4·2918707
6	1·3400956	1·4185191	26	3·5556726	4·7493829
7	1·4071004	1·5036302	27	3·7334563	4·8223459
8	1·4774554	1·5938480	28	3·9201291	5·1116867
9	1·5513282	1·6894789	29	4·1161356	5·4183879
10	1·6288946	1·7908476	30	4·3219423	5·7434912
11	1·7103393	1·8982985	31	4·5380394	6·0881007
12	1·7958563	2·0121964	32	4·7649414	6·4533867
13	1·8856491	2·1329282	33	5·0031885	6·8405899
14	1·9799316	2·2609039	34	5·2533479	7·2510253
15	2·0789281	2·3965581	35	5·5160152	7·6860868
16	2·1828745	2·5403516	36	5·7918101	8·147252
17	2·2920183	2·6927727	37	6·0814069	8·6860871
18	2·4066192	·8543391	38	6·3854772	9·1542523
19	2·526950	3·0255995	39	6·7047511	9·7035074
20	2·6532977	3·2071355	40	7·0399887	10·2857178

TABLE SECOND.

Showing the preſent value of 1l. or D. 1 due at the end of any number of years, from 1 to 40.

years.	5 per ct	6 per ct	years.	5 per ct.	6 per ct.
1	·952381	·943396	21	358942	·294155
2	·90703	·889996	22	34185	·277505
3	·863838	·889619	23	·325571	·261797
4	·822702	·792093	24	·310068	·246978
5	·783526	·747258	25	305303	·232998
6	·746215	·70496	26	281241	·21981
7	·710681	·665057	27	·267848	·207368
8	·676839	·627412	28	·255094	·19563
9	644609	·594898	29	·242946	·184556
10	·613913	·558394	30	·231377	·17411
11	·584679	·562787	31	·220359	·164 55
12	·556837	·496969	32	·209866	154957
13	·530321	·468839	33	·199872	146186
14	·505068	·442301	34	·190355	137912
15	·481017	·417265	35	·18129	·130105
16	·458311	·393647	36	·172057	·122741
17	·436297	·371364	37	·164436	·115793
18	·415521	·350343	38	·156605	109182
19	·395734	·330513	39	·149148	·103002
20	·376889	·311804	40	142046	·097172

TABLE THIRD.

Showing the amount of 1l. or D.1 annuity for any number of years from 1 to 40.

ys.	5 per cent.	6 per cent.	ys	5 per cent.	6 per cent.
1	1·	1·	21	35·719252	39·992725
2	2·05	2·06	22	38·505214	43·392289
3	3·1525	3·1836	23	41·430475	46·995826
4	4·310125	4·374616	24	44·501999	50·815576
5	5·525631	5·637093	25	47·727009	54·86451
6	6·801913	6·975318	26	51·113454	59·156381
7	8·142008	8·398387	27	54·669126	63·705763
8	9·549109	9·897467	28	58·402583	68·528109
9	11·026564	11·491315	29	62·322712	73·639796
10	12·577892	13·180794	30	66·438847	79·058183
11	14·206787	14·971642	31	70·76079	84·801674
12	15·917126	16·86994	32	75·298829	90·889775
13	17·712983	18·882132	33	80·063771	97·343161
14	19·598632	21·015064	34	85·066959	104·183751
15	21·578563	23·275968	35	90·320307	111·434776
16	23·657492	25·672527	36	95·836323	119·120863
17	25·840366	28·212879	37	101·628139	127·268114
18	28·132385	30·905652	38	107·709546	135·904201
19	30·529004	33·759991	39	114·095025	145·058453
20	33·065954	36·78559	40	120·799774	154·761961

TABLE FOURTH.

Showing the preſent worth of 1l. or D.1 annuity, for any number of years from 1 to 40.

yrs.	5 per cent	6 per cent	yrs.	5 per cent	6 per cent.
1	0·95238	0·94339	21	12·82115	11·76407
2	1·85941	1·83339	22	13·163	12·04158
3	2·72325	2·67301	23	13·48807	12·30338
4	3·54595	3·4651	24	13·79864	12·55035
5	4·32948	4·21236	25	14·09394	12·78335
6	5·07569	4·96732	26	14·37518	13·00316
7	5·78637	5·58238	27	14·64303	13·21053
8	6·46321	6·20979	28	14·89813	13·40616
9	7·10782	6·80169	29	15·14107	13·59072
10	7·72173	7·36008	30	15·37245	13·76483
11	8·30641	7·88687	31	15·59281	13·92908
12	8·86325	8·38384	32	15·80268	14·08398
13	9·39357	8·85268	33	16 00255	14·22917
14	9·89864	9·29498	34	16·1929	14·36613
15	10·37966	9·71225	35	16·37419	14·49533
16	10·83777	10·10589	36	16·54685	14·61722
17	11 27407	10·47726	37	16·71129	14·73211
18	11·68958	10·8276	38	16·86789	14·84048
19	12 08532	11·15811	39	17·01704	14·9427
20	12·46221	11·46992	40	17·15909	15·03913

TABLE FIFTH.

The annuity which 1l. or D.1 will purchaſe for any number of years to come, from 1 to 40.

yrs.	5 per cent	6 per ct.	yrs.	5 per ct.	6 per ct.
1	1·05	1 06	21	·078	·085
2	·5378	·54544	22	·07597	·08303
3	·36721	·37411	23	·07414	·08128
4	·28201	·28859	24	·07247	07968
5	·23097	·23739	25	·07095	·07823
6	·19702	·20336	26	06956	·0769
7	·17282	·17913	27	·06829	·0757
8	·15473	·16103	28	·06712	·07459
9	·14069	·14702	29	·06604	·07358
10	·1295	.13587	30	·06505	·07272
11	·12039	·12679	31	·06413	·07179
12	·11282	·11927	32	·06328	·071
13	·10645	·11296	33	·06249	·07037
14	·10102	·10758	34	·06175	·06959
15	·09624	·10296	35	·06107	·06899
16	·09227	·09895	36	·06043	·06839
17	·0887	·09544	37	.05984	·06785
18	·08555	·09235	38	·05928	·06735
19	·08274	·08962	39	·05876	·06689
20	·08024	·08718	40	·05828	·06646

ALLIGATION

IS the method of mixing two or more ſimples of different qualities, ſo that the compoſition may be of a mean or middle quality: It conſiſts of two kinds, viz. Alligation Medial, and Alligation Alternate.

ALLIGATION MEDIAL

Is, when the quantities and prices of ſeveral things are given, to find the mean price of the mixture compounded of thoſe things.

RULE.

As the ſum of the quantities, or the whole compoſition, is to their total value; ſo is any part of the compoſition to its mean price or value.

EXAMPLES.

1. A Tobacconiſt would mix 60lb. of tobacco at 6d. per lb. with 50lb. at 1s. 40lb. at 1s. 6d and 30lb. at 2s. per lb.: What is 1lb. of this mixture worth?

	lb.		s.	d.		£	s.
	60	at	0	6	is	1	10
	50	—	1	0	—	2	10
	40	—	1	6	—	3	0
	30	—	2	0	—	3	0
Sum of the ſimples	180				Total	10	0

lb. £ lb.
As 180 : 10 :: 1
1
—
10
20
—
180)200(1s.
180
—
20
12
—
180)240(1 $\frac{1}{3}$d.
180
—
60
—

Ans. 1s. 1$\frac{1}{3}$d.

2. A farmer would mix 20 buſhels of wheat at D1 per buſhel, 16 buſhels of rye at 75c. per buſhel, 12 buſhels of barley at 50c. per buſhel, and 8 buſhels of oats at 40c. per buſhel: What is the value of one buſhel of this mixture?

Ans. 73c $5\frac{5}{7}$m.

3. A wine merchant mixes 12 gallons of wine at 75c. per gallon, with 24 gallons, at 90c. and 16 gallons at D1 10c.: What is a gallon of this compoſition worth?

Ans. 92c. 6m.

4. A goldſmith melted together 8oz. of gold of 22 carats fine, 1lb. 8oz. of 21 carats fine, and 10oz. of 18 carats fine: Pray what is the quality, or fineneſs of the compoſition?

$$\text{Ans. } \frac{\overline{8\times22}+\overline{20\times21}+\overline{10\times18}}{8+20+10}=20\tfrac{8}{19}\text{ carats fine.}$$

5. A refiner melts 5lb. of gold of 20 carats fine with 8lb. of 18 carats fine: How much alloy muſt be put to it to make it 22 carats fine?

$$22-\overline{\overline{5\times20}+\overline{8\times18}\div5+8}=3\tfrac{3}{13}$$

Anſwer. It is not fine enough by $3\frac{3}{13}$ carats, ſo that no alloy muſt be added, but more gold.

ALLIGATION ALTERNATE*

IS the method of finding what quantity of each of the ingredients, whoſe rates are given, will compoſe a mixture of a given rate: ſo that it is the reverſe of Alligation Medial, and may be proved by it.

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[*] By connecting the leſs rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities reſulting are ſuch, that there is preciſely as much gained by one quantity as is loſt by the other, and therefore the gain and loſs upon the whole are equal, and are exactly the propoſed rate.

In like manner, let the number of ſimples be what it may, and with how many ſoever. each one is linked, ſince it is always a leſs with a

CASE I.

The whole work of this caſe conſiſts in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities ſought.

Rule.

1. Place the ſeveral prices of the ſimples, being reduced to one denomination, in a column under each other, the leaſt uppermoſt, and ſo gradually downward, as they increaſe, with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each ſimple, or ingredient, which is leſs than that of the compound, with one or any number of thoſe, which are greater than the compound, and each greater rate or price with one or any number of the leſs.

3. Place the difference, between the mean price, or mixture rate, and that of each of the ſimples, oppoſite to the rates with which they are connected.

4. Then, if only one difference ſtand againſt any rate, it will be the quantity belonging to that rate; but if there be ſeveral, their ſum will be the quantity.

Examples.

1. A merchant has ſpices, ſome at 1s. 6d. per lb. ſome at 2s. ſome at 4s. and ſome at 5s. per lb.: How much of each ſort muſt he mix that he may ſell the mixture at 3s. 4d. per lb.?

greater than the mean price, there will be an equal balance of loſs and gain between every two, and conſequently an equal balance on the whole.

It is obvious from the rule, that queſtions of this ſort admit of a great variety of anſwers; for having found one anſwer, we may find as many more as we pleaſe, by only multiplying or dividing each of the quantities found, by 2, 3, 4, &c. the reaſon of which is evident; for if two quantities of two ſimples make a balance of loſs and gain, with reſpect to the mean price, ſo muſt alſo the double or triple, the half or third part, or any other ratio of theſe quantities, and ſo on *ad infinitum*.

If any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be nothing, as water mixed with wine, and alloy with gold and ſilver.

	lb.	lb.		s.	d.	
Mean rate 40d.	18	20	at	1	6	per lb.
	24	8	—	2		
	48	16	—	4		
	60	22	—	5		

	lb.	lb.		s.	d.	
40d.	18	8	at	1	6	per lb.
	24	20	—	2		
	48	22	—	4		
	60	16	—	5		

	d.		lb.		s.	d.	
40d	18	20+8	28	at	1	6	per lb.
	24	8	8	—	2	0	
	48	16+22	38	—	4	0	
	60	22	22	—	5	0	

	d.		lb.		s.	d.	
40d.	18	20	20	at	1	6	per lb.
	24	8+20	28	—	2	0	
	48	16	16	—	4	0	
	60	22+16	38	—	5	0	

	d.		lb.		s.	d.	
40d.	18	8	8	at	1	6	per lb.
	24	8+20	28	—	2	0	
	48	22+16	38	—	4	0	
	60	16	16	—	5	0	

	d.		lb.		s.	d.	
40d.	18	8+20	28	at	1	6	per
	24	20	20	—	2	0	
	48	22	22	—	4	0	
	60	22+16	38	—	5	0	

	d.		lb.		s.	d.	
40d	18	20+8	28	at	1	6	per lb.
	24	8+20	28	—	2	0	
	48	16+22	38	—	4	0	
	60	22+16	38	—	5	0	

Note. Theſe ſeven anſwers ariſe from as many various ways of linking the ſimples together.

2. *A merchant has Canary wine, at 3s. per gallon, Sherry at 2s 1d. and Claret at 1s. 5d. per gallon: How much of each ſort muſt he take to ſell it at 2s. 4d. per gallon?

$$\text{Mean rate 28d.}\left\{\begin{array}{l}36\\25\\17\end{array}\right. \begin{array}{l}3+11\\8\\8\end{array}\left|\begin{array}{llll}14 & \text{at} & 3s. & 0d.\\8 & - & 2 & 1\\8 & - & 1 & 5\end{array}\right\}\text{per gal.}$$

3. A goldſmith would mix gold of 19 carats fine, with ſome of 16, 18, 23, and 24 carats fine, ſo that the compound may be 21 carats fine: What quantitity of each muſt he take?

Ans. 5oz. of 16 carats fine. 5oz of 18, 5oz. of 19, 10 oz. of 23, and 10 oz of 24 carats fine.

4. It is required to mix ſeveral ſorts of wine, at 60c. 90c. and D.1 15c. per gallon, with water, that the mixture may be worth 75c. per gallon: Of how much of each ſort muſt the compoſition conſiſt?

Ans. 40 gallons of water, 15 galls. of wine at 60c. 15 galls. do. at 90c. and 75 galls. do. at D.1 15.

CASE. II.

When the rates *of* all *the* ingredients, *the* quantity *of but* one *of them, and the* mean rate *of the whole mixture are given, to find the ſeveral quantities of the reſt, in proportion to the quantity given.*

Rule.

Take the differences between each price, and the mean rate, and place them alternately, as in Caſe I. Then, as the difference ſtanding againſt that ſimple, whoſe quantity is given, is to that quantity, ſo is each of the other differences, ſeverally, to the ſeveral quantities required.

Examples.

1. A merchant has 40lb. of tea, at 6 ſhillings per lb. which he would mix with ſome at 5s. 8d. ſome at 5s. 2d. and ſome at 4s. 6d.: How much of each ſort muſt he take to mix with the 40lb. that he may ſell the mixture at 5s. 5d. per lb.

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* Note, the ſecond queſtion admits but of one way of linking, and ſo but of one anſwer; yet all numbers in the ſame proportion between themſelves, as the numbers which compoſe the anſwer, will likewiſe ſatisfy the condition of the queſtion.

$$65\left\{\begin{array}{l}54\\62\\68\\72\end{array}\right.\quad\begin{array}{r|l}7+3 & 10\text{lb.}\\3+7 & 10\\3+11 & 14\\11+3 & 14\end{array}$$

ſtands againſt the given quantity.

$$\text{As } 14:40::\left\{\begin{array}{lllll}10 & : & 28\tfrac{8}{14} & \text{at } 4\text{s.} & 6\text{d.}\\10 & : & 28\tfrac{8}{14} & \text{—}\ 5 & 2\\14 & : & 40 & \text{—}\ 5 & 8\end{array}\right\}\text{per lb.}$$

2. How much gold of 16, 20, and 24 carats fine, and how much alloy, muſt be mixed with 10 oz. of 18 carats fine, that the compoſition may be 22 carats fine.

Ans. 100z. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

Alternation Total.

CASE III.

When the rates *of the ſeveral ingredients, the* quantity *to be compounded, and the* mean rate *of the whole mixture are given, to find how much of each ſort will make up the quantity.*

Rule.

Place the differences between the mean rate, and the ſeveral prices alternately, as in Caſe I.; then, as the ſum of the quantities, or differences, thus determined, is to the given quantity, or whole compoſition; ſo is the difference of each rate, to the required quantity of each rate.

Examples.

1. Suppoſe I have four ſorts of currants, at 8d. 12d. 18d. and 22d. per lb.; the worſt will not ſell, and the beſt are too dear; I therefore conclude to mix 120lb. and ſo much of each ſort as to ſell them at 16d. per lb.: How much of each ſort muſt I take?

$$\begin{array}{l}\\16\text{d.}\left\{\begin{array}{l}8\\12\\18\\22\end{array}\right.\\ \\ \text{Sum}=\end{array}\begin{array}{l}\text{d.}\quad\text{lb.}\\ \begin{array}{ll}&6\\&2\\&4\\&8\end{array}\\ \hline \quad 20\end{array}\qquad \begin{array}{c}\text{lb.}\quad\text{lb.}\\ \text{As } 20:120::\end{array}\left\{\begin{array}{lllll}\text{lb.}&&\text{lb.}&&\text{d.}\\6&:&36&\text{at}&8\\2&:&12&\text{—}&12\\4&:&24&\text{—}&18\\8&:&48&\text{—}&22\\ \hline &&120&&\end{array}\right\}\text{per lb.}$$

2. A goldſmith has ſeveral ſorts of gold; viz. of 16, 17, 20 and 22 carats fine, and would melt together, of all

theſe ſorts, ſo much as may make a maſs of 40oz. 12 carats fine; how much of each ſort is required?

Ans. 16oz. 15 carats fine, 8oz. 17, 4oz. 20, and 12oz. of 22 carats fine.

3. A merchant would mix four ſorts of wine, of ſeveral prices, viz. at 75c. D.1 25c. D.1 50c. and D 1 62½c per gallon; of theſe he would have a mixture of 72 gallons, worth D.1 37½c per gallon; what quantity of each ſort muſt he have.

Ans. 8 at 75c. 16 at D1 25c. 40 at D.1 50c. and 8 at D.1 62½c. Or, 16 at 75c. 8 at D.1 25c. 8 at D.1 5cc. and 40 at D.1 62½c.

CASE IV.

When more than one of the Simples are limited.

Rule.

Find, by Alligation Medial, what will be the *rate* of a mixture made of the *given quantities* of the *limited ſimples* only; then, conſider this as the *rate* of a *limited ſimple*, whoſe *quantity* is the ſum of the firſt given *limited ſimples*, from which, and the *rates* of the *unlimited ſimples*, by Caſe 2d. calculate the quantity.

Examples.

1. How much wine at 80c. and at 87½c. per gallon, muſt be mixed with 8 gallons at 75c. and 12 gallons at 90c. per gallon, that the mixture may be worth 82½c. per gallon?

$$\text{Limited ſimples}\left\{\begin{array}{rlcr} 8 & \text{gallons, at } 75c. & = & D.\ 6 \\ 12 & \text{gallons, at } 90 & = & 10\ 80c. \\ \hline 20 & & & 16\ 80 \end{array}\right\}$$

$$\begin{array}{lllll} & \text{Gal.} & \text{D. c.} & \text{Gal.} & \text{c.} \\ \text{As} & 20 : & 16\ 80 :: & 1 : & 84 \text{ per gallon.} \end{array}$$

Now, having found the rate of the limited ſimples, the queſtion may ſtand thus: How much wine, at 80c. and 87½c. per gallon muſt be mixed with 20 gallons at 84c. per gallon, that the mixture may be worth 82½c per gallon?

$$82\tfrac{1}{2}\left\{\begin{array}{l} 80 \\ 84 \\ 87\tfrac{1}{2} \end{array}\right. \quad \begin{array}{r|l} 1\tfrac{1}{2}+5 & 6\tfrac{1}{2} \text{ gallons at } 80c. \\ 2\tfrac{1}{2} & 2\tfrac{1}{2} \qquad - \ 84 \\ 2\tfrac{1}{2} & 2\tfrac{1}{2} \qquad - \ 87\tfrac{1}{2} \end{array}$$

$$\text{As } 2\tfrac{1}{2} : \begin{Bmatrix} 6\tfrac{1}{2} \\ 2\tfrac{1}{2} \end{Bmatrix} :: 20 : \begin{Bmatrix} \text{52 galls. at 80c. per gall.} \\ \text{20 ——— } 87\tfrac{1}{2}\text{c. ———} \end{Bmatrix} \text{Ans.}$$

Proof.

52	gallons at	80c.	=	D.41	60c.
50	———	87½	=	17	50
8	———	75	=	6	
12	———	90	=	10	80
92	———	82½	=	75	90

POSITION.

POSITION is a rule, which, by falſe or ſuppoſed numbers, taken at pleaſure, diſcovers the true ones required. It is divided into two parts ; *ſingle* and *double*.

SINGLE POSITION.

Single Poſition teaches to reſolve thoſe queſtions, whoſe reſults are proportional to their ſuppoſitions ; ſuch are thoſe which require the multiplication or diviſion of the number ſought by any propoſed number ; or when it is to be increaſed or diminiſhed by itſelf a certain propoſed number of times.

Rule.

1. Take any number, and perform the ſame operations with it as are deſcribed to be performed in the queſtion.

2. Then ſay, as the ſum of the errours is to the given ſum, ſo is the ſuppoſed number to the true one required.

Proof. Add the ſeveral parts of the ſum together, and if it agrees with the ſum, it is right.

Examples.

1. A ſchool-maſter, being aſked how many ſcholars he had, ſaid, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I ſhould then have 435 : Of what number did his ſchool conſiſt ?

Suppoſe he had 80	As 290 : 435 :: 80	
As many=80	80	120
$\frac{3}{4}$ as many=60	——	120
$\frac{1}{2}$ as many=40	29|0)3480|0(120 Ans.	90
$\frac{1}{4}$ as many=20	29	60
$\frac{1}{8}$ as many=10	——	30
——	58	15
290	58	——
	——	Proof 435
	0	

2. A perſon lent his friend a ſum of money unknown, to receive intereſt for the ſame at 6 per cent. per annum, ſimple intereſt, and at the end of 12 years, received for principal and intereſt D860 : What was the ſum lent ?

Ans. D500.

3. A, B and C joined their ſtocks, and gained D353 12$\frac{1}{2}$c. of which A took up a certain ſum, B took up four times ſo much as A, and C, five times ſo much as B : What ſhare of the gain had each?

Ans.	D 14	12$\frac{1}{2}$c.	A's ſhare.
	56	50	B's ſhare.
	282	50	C's ſhare.

4. A, B, C and D ſpent 35s. at a reckoning, and, being a little dipped, they agreed that A ſhould pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, D $\frac{1}{4}$: What did each pay in the above proportion ?

Ans	A	13s.	4d.
	B	10	6
	C	6	8
	D	5	0

5. A certain ſum of money is to be divided between 5 men, in ſuch a manner as that A ſhall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is £40 : What is the ſum ?

Suppoſe £200. Then $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}=120$.

200—120=80. As 80 : 40 :: 200 : 100 Ans.

6. A and B, talking of their ages, B ſaid his age was once and an half the age of A ; C ſaid his was twice and one tenth the age of both, and that the ſum of their ages was 93 : What was the age of each ?

Ans. A's 12, B's 18, and C's 63 year

7. A and B having found a purſe of money diſputed who ſhould have it: A ſaid that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to £35, and, if B could tell him how much was in it he ſhould have the whole, otherwiſe he ſhould have nothing: How much did the purſe contain?

Ans. £100.

8. A gentleman divided his fortune among his ſons; to A he gave D9 as often as to B D5, and to C but D3 as often as to B D7, yet C's portion came to D1059: What was the whole eſtate? Ans. D7979 80c.

9. Seven eighths of a certain number exceeds four fifths by 6: What is that number? Ans. 80.

DOUBLE POSITION.

Double Poſition teaches how to reſolve queſtions by making two ſuppoſitions of falſe numbers.

Thoſe queſtions, in which the reſults are not proportional to their poſitions, belong to this rule: ſuch are thoſe, in which the number ſought is increaſed or diminiſhed by ſome given number, which is no known part of the number required.

RULE.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the queſtion.

2. Place the reſult or errours againſt their poſitions or ſuppoſed numbers, thus,

$$\begin{array}{ccc} \text{Pos.} & & \text{Err.} \\ 30 & \times & 12 \\ 20 & & 6 \end{array}$$

and if the errour be too great, mark it with +; and if too ſmall with —.

2. Multiply them croſſwiſe; that is, the firſt poſition by the laſt errour, and the laſt poſition by the firſt errour.

4. If the errours be alike, that is, both too ſmall or both too great, divide the difference of the products by the difference of the errours, and the quotient will be the anſwer.

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* The rule is founded on this ſuppoſition, that the firſt errour is to the ſecond as the difference between the true and firſt ſuppoſed number is to the difference between the true and ſecond ſuppoſed number: When that is not the caſe, the exact anſwer to the queſtion cannot be found by this rule.

5. If the errours be unlike, that is, one too ſmall, and the other too great, divide the ſum of the products by the ſum of the errours, and the quotient will be the anſwer.

Note. When the errours are the ſame in quantity, and unlike in quality, half the ſum of the ſuppoſitions is the number ſought.

EXAMPLES.

1. A lady bought damaſk for a gown, at 8s. per yard, and lining for it at 3s. per yard; the gown and lining contained 15 yards, and the price of the whole was 3l. 10s.: How many yards were there of each?

Suppoſe 6 yards damaſk, value 48s.
Then ſhe muſt have 9 yards lining, value 27s.

Sum of their values=75s.
So that the firſt errour is 5 too much, or + 5
Again, ſuppoſe ſhe had 4 yards of damaſk, value 32s.
Then ſhe muſt have 11 yards of lining, value 33s.

Sum of their values=65s.
So that the ſecond errour is 5 too little, or — 5

Then 6 X 5+   5 yards at 8s=£2 0 0
4 X 5—   10yds at 3s.= 1 10 0

20   30   Proof 3 10 0
20

Sum of errours=5+5=10)50

Ans. 5yds. damaſk, and 15—5=10 yards lining. Or 6+4÷2=5 as before.

2. A labourer was hired for 60 days, upon this condition, that, for every day he wrought, he ſhould receive 75c.; and for every day he was idle, ſhould forfeit 37½c, at the expiration of the time he received D18: How many days did he work, and how many was he idle?

Suppoſe he worked {20 X 18—
40 X 4½+

Ans. He was employed 36 days, and was idle number of quantities to be taken at a time; the product of all the terms will be the anſwer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array} \qquad \text{Or, } 8\times7\times6\times5 \ (=4 \text{ terms}) =1680 \text{ Ans.}$$

2. How many words can be made with 6 letters of the alphabet, admitting a number of conſonants may make a word?

$24\times23\times22\times21\times20\times19$ (6 terms) $=96909120$, Ans.

PROBLEM III.

Any number of things being given, whereof there are ſeveral things of one ſort, ſeveral of another, &c. to find how many changes may be made out of them all.

RULE.[*]

1. Take the ſeries $1\times2\times3\times4$, &c. up to the number of things given, and find the product of all the terms.

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[*] Any 2 quantities, *a*, *b*, both different, admit of 2 changes; but if the quantities are the ſame, or *ab* becomes *aa* there will be only one alteration, which may be expreſſed by $\frac{1\times2}{1\times2}=1$.

Any 3 quantities, *a*, *b*, *c*, all different from each other, admit of 6 variations; but if the quantities are all alike, or *abc* become *aaa*, then the 6 variations will be reduced to 1, which may be expreſſed by $\frac{1\times2\times3}{1\times2\times3}=1$. Again, if 2 quantities out of 3 are alike, or *abc* become *aac*; then the 6 variations will be reduced to theſe 3, *aac*, *caa*, *aca*, which may be expreſſed by $\frac{1\times2\times3}{1\times2}=3$, and ſo of any greater number.

2. Take the ſeries $1\times2\times3\times4$, &c. up to the number of the given things of the firſt ſort, and the ſeries, $1\times2\times3\times4$, &c. up to the number of the given things of the ſecond ſort, &c.

3. Divide the product of all the terms of the firſt ſeries by the *joint* product of all the terms of the remaining ones, and the quotient will be the anſwer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15$ (= number of letters in the word) $=1307674368000$.

$1\times2\times3\times4\times5$ (=number of *a*s) $=120$
1×2 (=number of *p*s) $=2$
1 (=number of *t*s) $=1$
$1\times2\times3$ (=number of *h*s) $=6$
1×2 (=number of *n*s) $=2$

$2\times6\times1\times2\times120=2880$)1307674368000(454053600 Ans.

2. How many different numbers can be made of the following figures, 1223334444? Ans. 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.*

1. Take the ſeries 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a ſeries of as many terms, decreaſing by 1 from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the laſt product by the former, and the quotient will be the number ſought.

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* In any given number of quantities, the number of Combinations increaſes gradually till you come about the mean numbers, and then gradually decreaſes. If the number of quantities be *even*, half the number of places will ſhew the greateſt number of Combinations that can be made of thoſe quantities; but if odd, then thoſe two numbers which are the middle, and whoſe ſum is equal to the given number of quantities, will ſhew the greateſt number of Combinations.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12 ?

$1\times2\times3\times4\times5\times6\times7$ (= the number to be taken at a time, $=5040$.

$12\times11\times10\times9\times8\times7\times6$(= ſame number from 12)$=3991680$.

5040)3991680(792 Ans.

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet ?

Ans. 134596.

3. A farmer bargained with a gentleman for a dozen ſheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but being long chooſing them, the gentleman told him that if he would give him a cent for every different dozen which might be choſen out of the two dozen, he ſhould have the whole, to which the farmer readily agreed : Pray what did they coſt him ?

Ans. D27041 56c.

PROBLEM V.

To find the compoſitions of any number, in an equal number of ſets, the things being all different.

RULE.

Multiply the number of things in every ſet continually together, and the product will be the anſwer required.

EXAMPLES.

1. Suppoſe there are five companies, each conſiſting of 9 men ; it is required to find how many ways 5 men may be choſen, one out of each company ?

Multiply 9 into itſelf continually, as many times as there are companies.

$9\times9\times9\times9\times9=59049$ different ways, Ans.

2. How many changes are there in throwing 4 dice ?

As a die has 6 ſides, multiply 6 into itſelf four times continually. $6\times6\times6\times6=1296$ changes, Ans.

3. In how many ways may a man, a woman and a child be choſen out of three companies, conſiſting of 5 men, 7 women and 9 children ? Ans. 315.

THE USE OF LOGARITHMS.

1. In Multiplication.

Given two numbers, viz. 275 and 12·6 to find their product.

Rule. To the logarithm of 275, viz.	2·43933
Add the logarithm, of 12·6, viz.	1·10037
And their ſum is the logarithm of their product, viz.	3465=3·53970

2. In Division.

Let it be required to find the quotient, which ariſes by dividing one number by another ; ſuppoſe 1425 by 57.

From the logarithm of the dividend, viz.	1425=3·15381
Take the logarithm of the diviſor, viz. 57	=1·75587
And the remainder is the logarithm of the quotient, viz.	25=1·39794

3. In the Rule of Three.

Three numbers given to find a fourth, in direct proportion.

Rule. From the tables take the logarithms of each of the propoſed numbers, then add the logarithms of the ſecond and third together, and from the ſum take the logarithm of the firſt and the remainder will be the logarithm of the fourth number.

Let the three propoſed numbers be 18, 24 and 33, and the operation will ſtand thus :

1·38021 = the logarithm of 24, the 2d term.
1·51851 = the logarithm of 33, the 3d term.

2·89872 = the logarithm of their product.
—1·25527 = the logarithm of the 1ſt term 18.

1·64345 = the logarithm of the 4th term required which, by the Table, anſwers to the natural number 4 the 4th proportional to the three propoſed numbers.

4. IN INVOLUTION, OR RAISING POWERS.

To find any power of any propoſed number, or to involve any number to any propoſed power, by logarithms.

RULE. Multiply the logarithm of the given root by the power, viz. by 2 for the ſquare, by 3 for the cube, &c. and the product is the logarithm of the power ſought.

Required to find the cube of 12?

1·07918 = the logarithm of 12.
× 3 = the third power, or cube.

3·23754 = 1728 the cube of 12.

5. IN EVOLUTION, OR EXTRACTING ROOTS.

To extract any root of any propoſed number.

RULE. Divide the logarithm of the propoſed number by the index of the required root, viz. by 2 for the ſquare, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required to find the cube root of 1728?

3·23754 = the logarithm of 1728, and 3·23754 ÷ 3 = 1·07918 is the logarithm of the cube root of 1728, viz. 12.

6. IN COMPOUND INTEREST.

To find the amount of any ſum for any time and at any rate, at compound intereſt.

RULE. Multiply the logarithm of the ratio (i. e. the amount of £1 or D1 for 1 year) by the number of years, and to the product add the logarithm of the principal; the ſum will be the logarithm of the amount.

What will 45l. amount to forborne 12 years, at 6 per cent. per annum, compound intereſt?

Log. of 1·06, the ratio is ·02533
Multiply by the time 12

·30396
Add Log. of 45 the principal 1·65321

The ſum is 1·95717 which is the logarithm of 90·7 = £90 14s. Ans.

7. IN DISCOUNT AT COMPOUND INTEREST.

To find the preſent worth of any ſum of money, due any time hence, at any rate, at compound intereſt.

RULE. From the logarithm of the ſum to be diſcounted, ſubtract the logarithm of the rate multiplied by the time ; and the remainder is the logarithm of the preſent worth.

What preſent money will pay a debt of 90l. 14s. due 12 years hence, diſcounting at the rate of 6 per cent. per annum ?

From the logarithm of 90l. 14=1·95717

Sub. product of the Log. of the ratio × by the time, } = ·30396

The remainder 1·65321

is the logarithm of 45l. Ans.

RULES

For reducing Federal Money to the ſeveral former currencies of the United States and to Engliſh Money, and the contrary.

	s.	d.	
D1=	6	0	old currency of New England and Virginia.
	8	0	New York and N. Carolina.
	7	6	New Jerſey, Pennſylvania, Delaware and Maryland.
	4	8	South Carolina and Georgia.
	4	6	Sterling or Engliſh Money.

NOTE. Moſt of the operations under the following rules are by decimals ; ſo that, if pounds with lower denominations be given, the lower denominations muſt be reduced to decimals of a pound ; and, if dollars, &c. be reduced to pounds and decimals, the decimals may be reduced to ſhillings, &c. in the anſwer.

I. *To reduce Federal Money and the old currency of New-England and Virginia to each other.*

Multiply dollars by ·3, or divide by $3\frac{1}{3}$ for pounds.
Multiply pounds by $3\frac{1}{3}$, or divide by ·3 for dollars.

EXAMPLES.

1. Reduce D150 to pounds.

·3

£45·0 Ans.

Or, $3\frac{1}{3}=\frac{10}{3}$; and, $150\div\frac{10}{3}=150\times\frac{3}{10}=\frac{450}{10}=$ £45 Ans. as before.

2. Reduce £45 to dollars.

$3\frac{1}{3}$

135
15

D150 Ans.

Or, ·3)45·

D150· Ans. as before.

3. Reduce D17 81c. $2\frac{1}{2}$m. to pounds, &c.

17·8125
·3

£5·34375
20

s. 6·875
12

d. 10·5
4

qr. 2·0

Ans. £5 6s. $10\frac{1}{2}$d.

Or, $3\frac{1}{3}=\frac{10}{3}$; and 17·8125 $\div\frac{10}{3}=17{\cdot}8125\times\frac{3}{10}$, or ×·3; hence the operation and reſult as before,

4. Reduce £5 6s. $10\frac{1}{2}$d. to dollars, &c.

4 | 2· qrs.
12 | 10·5 d.
20 | 6·875 s.

£5·34375
$3\frac{1}{3}$

16·03125
1·78125

D17·8125 Ans.

Or, ·3)5·34375

D17·8125 Ans. as before.

5. Reduce D271 17c. 5m. to pounds, &c.
Ans. £81 7s. $0\frac{1}{2}$d.+

6. Reduce £81. 7s. $0\frac{1}{2}$d. to dollars, &c.
Ans. D271 17c. 5m.—

7. Reduce D138 62c. 5m. to pounds, &c.
Ans. £41 11s. 9d.

8. Reduce £41 11s. 9d. to dollars, &c.
Ans. D138 62c. 5m.

II. *To reduce Federal Money to the old currency of New-York and North-Carolina, and the contrary.*

Multiply dollars by ·4, or divide by 2·5, for pounds.

Multiply pounds by 2·5, or divide by ·4 for dollars.

EXAMPLES.

1. Reduce 59 dollars to pounds, &c.

59
·4
£23·6
20
s. 12·
Ans. £23 12s.

£ s.
Or, 2·5)59·0(23 12 Ans.
50 as before.
90
75
15
20
300
25
50
50

2. Reduce £23 12s. to dollars, &c.

£23 12s.=£23·6
2·5
1180
472
D59·80 Ans.

Or, ·4)23·6
D59· Ans. as before.

3. Reduce D212 43c. 7$\frac{1}{2}$m. to pounds, &c.

D212·4375×·4=£84·975=£84 19s. 6d. Ans.

Or, 212·4375÷2·5=£84·975=£84 19s. 6d. Ans. as before.

4. Reduce £84 19s. 6d. to dollars, &c.

£84 19s. 6d =£84·975
£84·975×2·5 = D212·4375 Ans.

Or, £84·975÷·4=D212·4375 Ans. as bef

5. Reduce D74 $33\frac{1}{3}$c. to pounds, &c.

Ans. £29 14s. 8d.

6. Reduce £29 14s. 8d. to dollars, &c.

Ans. D74 $33\frac{1}{3}$c.

III. *To reduce Federal Money and the old currency of New-Jerſey, Pennſylvania, &c. to each other.*

Multiply dollars by ·375, or divide by $2\frac{2}{3}$, for pounds.
Multiply pounds by $2\frac{2}{3}$, or divide by ·375 for dollars.

EXAMPLES.

1. Reduce D59 75c. to pounds, &c.

D59·75
·375

29875
41825
17925

£22·40625 = £22 8s. $1\frac{1}{2}$d. Ans.

Or, $2\frac{2}{3}=\frac{8}{3}$; and D59·75 $\div\frac{8}{3}=59\ 75\times\frac{3}{8}=\frac{179{\cdot}25}{8}=$ £22·40625 as before.

2. Reduce £22 8s. $1\frac{1}{2}$d. to dollars, &c.

£22 8s. $1\frac{1}{2}$d.=£22·40625
$2\frac{2}{3}$

44·81250
14·9375

Ans. D59·75

Or,

·375)22·40625(D59·75
Ans. as before.

3. Reduce D79 87c. 5m. to pounds &c.

Ans. £30 19s. $0\frac{3}{4}$d.

4. Reduce £30 19s. $0\frac{3}{4}$d. to dollars, &c.

Ans. D79 87c. 5m.

5. Reduce D131 80c. to pounds, &c.

Ans. £49 8s. 6d.

6. Reduce £49 8s. 6d. to dollars, &c.

Ans. D131 80c.

IV. *To reduce Federal Money and the old currency of South-Carolina and Georgia to each other.*

Multiply dollars by $\cdot2\frac{1}{3}$, or divide by $4\frac{2}{7}$, for pounds.
Multiply pounds by $4\frac{2}{7}$, or divide by $\cdot2\frac{1}{3}$, for dollars.

EXAMPLES.

1. Reduce 375 dollars to pounds, &c.

$$\begin{array}{r} 375\cdot \\ \cdot 2\tfrac{1}{3} \\ \hline 75\cdot 0 \\ 12\cdot 5 \\ \hline £87\cdot 5 \\ 20 \\ \hline \text{s. } 10\cdot \end{array}$$

Or, $4\frac{2}{7}=\frac{30}{7}$; and $375\div\frac{30}{7}=375\times\frac{7}{30}=\frac{2625}{30}=87\cdot 5$ as before.

2. Reduce £87 10s. to dollars, &c.

$$\begin{array}{r} £87\ 10\text{s.}=£87\cdot 5 \\ 4 \\ \hline 350\cdot \\ 12\cdot 5=\tfrac{1}{7} \\ 12\cdot 5=\tfrac{1}{7} \\ \hline \text{D}375\cdot\ \text{Ans.} \end{array}$$

Or, $\cdot 2\frac{1}{3}=\frac{\cdot 7}{3}$; and $£87\cdot 5\div\frac{\cdot 7}{3}=£87\cdot 5\times\frac{3}{\cdot 7}=\frac{262\cdot 5}{\cdot 7}$ $=$D375· Ans. as before.

3. Reduce D67 87c. 5m. to pounds, &c.
Ans. £15 16s. 9d.

4. Reduce £15 16s. 9d. to dollars, &c.
Ans. D67 87c. 5m.

5. Reduce D100 to pounds, &c. Ans. £23 6s. 8d.

6. Reduce £23 6s. 8d. to dollars, &c. Ans. D100.

V. *To reduce Federal Money to English Money, and the contrary.*

Multiply dollars by ·225, or divide by $4\frac{4}{9}$ for pounds.
Multiply pounds by $4\frac{4}{9}$, or divide by ·225, for dollars.

EXAMPLES.

1. Reduce D444 $44\frac{4}{9}$c. to pounds.

$$\begin{array}{r} \text{D}444\ 44\tfrac{4}{9} \\ \cdot 225 \\ \hline 222220 \\ 88888 \\ 88888 \\ 100=\tfrac{4}{9}\text{ of }\cdot 225 \\ \hline £100\cdot 00000\ \text{Ans.} \end{array}$$

Or, $4\frac{4}{9}=\frac{40}{9}$; and $444{\cdot}44\frac{4}{9}=\frac{4000}{9}$; then, $\frac{4000}{9} \div \frac{40}{9} = \frac{4000}{9} \times \frac{9}{40} = \frac{36000}{360}$ =£100 Ans. as before.

2. Reduce £100 to dollars, &c.

$4\frac{4}{9}$

400

$11{\cdot}11\frac{1}{9} = \frac{1}{9}$

$44{\cdot}44\frac{4}{9} = \frac{4}{9}$

D$444{\cdot}44\frac{4}{9}$ Ans.

Or, ·225)100·(D$444{\cdot}44\frac{4}{9}$ Ans. as before.

3. Reduce D25 to pounds, &c.

25×·225=5·625 ; and £5·625=£5 12s. 6d. Ans.

Or, $\frac{25}{4\frac{4}{9}}$ = 5·625, as before.

4. Reduce £5 12s. 6d. to dollars.

£5 12s. 6d.=£5·625 ; and 5·625×$4\frac{4}{9}$=D25 Ans.

Or, ·225)5·625(D25 Ans. as before.

5. Reduce D36 93c. $7\frac{1}{2}$m. to pounds, &c.

Ans. £8 6s. 2d. $2\frac{1}{2}$qr.

6. Reduce £8 6s. 2d. $2\frac{1}{2}$qr. to dollars, &c.

Ans. D36 93c. $7\frac{1}{2}$m.

A TABLE directing how to buy and ſell by the hundred, in Federal Money.

D.	D.	D.	D.	D.	D.
D.d.	D.d.	Dd.	D.d.	Dd.	D.d.
d. c.	D.d.c.	d. c.	D. d.c.	d.c.	D.d.c.
c.m.	D.dcm.	cm	D.dcm.	cm.	D.dcm.
1	112	34	3808	67	7504
2	224	35	3920	68	7616
3	336	36	4032	69	7728
4	448	37	4144	70	7840
5	560	38	4256	71	7952
6	672	39	4368	72	8064
7	784	40	4480	73	8176
8	896	41	4592	74	8288
9	1008	42	4704	75	8400
10	1120	43	4816	76	8512
11	1232	44	4928	77	8024
12	1344	45	5040	78	8736
13	1456	46	5152	79	8848
14	1568	47	5264	80	8960
15	1680	48	5376	81	9072
16	1792	49	5488	82	9184
17	1904	50	5600	83	9296
18	2016	51	5712	84	9408
19	2128	52	5824	85	9520
20	2240	53	5936	86	9632
21	2352	54	6048	87	9744
22	2464	55	6160	88	9856
23	2576	56	6272	89	9968
24	2688	57	6384	90	10080
25	2800	58	6496	91	10292
26	2912	59	6608	92	10304
27	3024	60	6720	93	10416
28	3136	61	6832	94	10528
29	3248	62	6944	95	10640
30	3360	63	7056	96	10752
31	3472	64	7168	97	10864
32	3584	65	7280	98	10976
33	3696	66	7392	99	11088

A TABLE for reducing Troy Weight to Avoirdupois.

Troy.	Avoir.	Troy.	Avoirdupois.		
gr.	drams.	oz.	lb.	oz.	drams.
1	·04	1		1	1·55
2	·07	2		2	3·11
3	·11	3		3	4·66
4	·15	4		4	6·22
5	·18	5		5	7·77
6	·[illegible]	6		6	9·32
7	·26	7		7	10·89
8	·29	8		8	12·44
9	·33	9		9	14·
10	·36	10		10	15·56
11	·4	11		12	1·09
12	·44	lb.			
13	·47	1	0	13	2·65
14	·51	2	1	10	5·3
15	·55	3	2	7	8·
16	·58	4	3	4	10·6
17	·62	5	4	1	13·25
18	·66	6	4	14	15·9
19	·69	7	5	12	2·56
20	·73	8	6	9	5·21
21	·77	9	7	6	7·86
22	·8	10	8	3	10·52
23	·84	20	16	7	5·03
pwt.		30	24	10	15·54
1	0·88	40	32	14	10·05
2	1·75	50	41	2	4·57
3	2·63	60	49	5	15·08
4	3·51	70	57	9	9·6
5	4·39	80	65	13	4·11
6	5·27	90	74	0	13·62
7	6·14	100	82	4	9·15
8	7·02	200	164	9	2·28
9	7·9	300	246	13	11·42
10	8·78	400	329	2	4·57
11	9·65	500	411	6	13·71
12	10·53	600	493	11	6·85
13	11·41	700	576	0	0·
14	12·29	800	658	4	9·14
15	13·16	900	740	9	2·28
16	14·04	1000	822	13	11·42
17	14·92	2000	1645	11	6·84
18	15·79	3000	2528	9	2·26
19	16·67	4000	3291	6	13·68

A TABLE for reducing Avoirdupois Weight to Troy Weight.

Av.	*Troy.*				*Avoir.*	*Troy.*			
dr.	lb.	*oz.*	*pwt.*	*gr.*	lb.	lb.	*oz.*	*pwt.*	*gr.*
½				13·67	1	1	2	11	16
¾				20·51	2	2	5	3	8
1			1	3·34	3	3	7	15	0
2			2	6·68	4	4	10	6	16
3			3	10·02	5	6	0	18	8
4			5	13·36	6	7	3	10	0
5			6	16·7	7	8	6	1	16
6			7	20·04	8	9	8	13	8
7			8	23·38	9	10	11	5	0
8			9	2·72	10	12	1	16	16
9			10	6·06	20	24	3	13	8
10			11	9·4	30	36	5	10	0
11			12	12·74	40	48	7	6	16
12			13	15·08	50	60	9	3	8
13			14	19·42	60	72	11	0	0
14			15	22·76	70	85	0	16	16
15			17	2·1	80	97	2	13	8
oz.					90	109	4	10	0
1			18	5·5	100	121	6	6	16
2		1	16	11	200	243	0	13	8
3		2	14	16·5	300	364	7	0	0
4		3	12	22	400	486	1	6	16
5		4	11	3·5	500	607	7	13	8
6		5	9	9	600	729	2	0	0
7		6	7	14·5	700	850	8	6	16
8		7	5	20	800	972	2	13	8
9		8	4	1·5	900	1093	9	0	0
10		9	2	7	1000	1215	3	6	16
11		10	0	12·5	2000	2430	6	13	8
12		10	18	18	3000	3645	10	0	0
13		11	16	23·5	4000	4861	1	6	16
14	1	0	15	5	5000	6076	4	13	8
15	1	1	13	10·5	6000	7291	8	0	0

By a law of the Congreſs of the United States, foreign gold and ſilver coins are made a legal tender for the payment of all debts and demands, at the ſeveral and reſpective rates following, viz. The gold coins of Great-Britain and Portugal, of their preſent ſtandard, at the rate of 100 cents for every 27 grains of the actual weight thereof—Thoſe of France and Spain 27$\frac{2}{3}$ grains of the actual weight thereof—Spaniſh milled dollars, weighing 17pwt. 7grs. equal to 100 cents, and in proportion for the parts of a dollar—Crowns of France, weighing 18pwt. 17grs. equal to 110 cents, and in proportion for the parts of a crown. It is alſo enacted, That every cent ſhall contain 208 grains of copper, and every half cent 104 grains; agreeably to which the following Tables are calculated.

A TABLE of the Weights of ſeveral pieces of Engliſh, Portugueſe, and French Gold Coins, according to an act of Congreſs, paſſed Nov. 1792.

	Pwt.	Gr.	Dols.	Cents.	Mills.
Double Johannes	18		16	0	0
Single ditto	9		8	0	0
Engliſh Guinea	5	6	4	66$\frac{2}{3}$	0
Half ditto	2	15	2	33$\frac{1}{3}$	0
French Guinea	5	6	4	59	8
Half ditto	2	15	2	29	9
4 Piſtoles	16	12	14	45	2
2 Piſtoles	8	6	7	22	6
1 Piſtole	4	3	3	61	3
Moidore	6	22	6	14	8

A TABLE of the value of Engliſh and Portugueſe Gold, in Dollars, Cents, and Mills, throughout the United States.

Gr.	Cts.	m.	Pwt.	Dols.	Cts.
1	3	7	1	0	89
2	7	4	2	1	77¾
3	11	1	3	2	66⅓
4	14	8	4	3	55½
5	18½	0	5	4	44
6	22	2	6	5	33⅓
7	25	9	7	6	22
8	29	6	8	7	11
9	33⅓	0	9	8	0
10	37	0	10	8	89
11	40	7	11	9	77¾
12	44	4	12	10	66⅔
13	48	1	13	11	55½
14	51	8½	14	12	44½
15	55	5½	15	13	33⅓
16	59¼	0	16	14	22
17	63	0	17	15	11
18	66⅔	0	18	16	0
19	70	4	19	16	89
20	74	0	1oz	17	77¾
21	77¾	0			
22	81½	0			
23	85	2			

NOTE. 89 cents is the value of 1 pennyweight of Engliſh and Portugueſe gold.

A TABLE of the value of French and Spaniſh Gold, in Dollars, Cents, and Mills, throughout the United States.

Gr.	Cts.	m.	Pwt.	Dols.	Cts.	m.
1	3	6	1	0	87	6
2	7	3	2	1	75	2
3	11	0	3	2	62¾	0
4	14	6	4	3	50	3
5	18	2	5	4	38	0
6	21	9	6	5	25	5
7	25	5	7	6	13	1
8	29	2	7	7	0	7
9	32	8	8	7	88	3
10	36½	0	9	8	76	0
11	40	1	10	9	63½	0
12	43	8	11	10	51	1
13	47	4	12	11	38	7
14	51	2	13	12	26	3
15	54¾	0	14	13	13	9
16	58	4	14	14	1½	0
17	62	0	15	14	89	0
18	65	7	16	15	76	6
19	69	3	17	16	64	2
20	73	0	1 oz	17	51	8
2	76	6				
22	80	3				
23	83	9				

NOTE. 87 cents 6 mills is the value of 1 pennyweight French and Spaniſh Gold.

FINIS.

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